Simplifying transducers

removing two-wayness and non-determinism

Pierre-Alain Reynier

LIF, Aix-Marseille Université & CNRS

Based on joint works with L Daviaud, E Filiot, O Gauwin, I Jecker, B Monmege, F Servais, JM Talbot and D Villevalois

Genesis of this work

- PhD at LSV on timed systems with Patricia Bouyer and François Laroussinie

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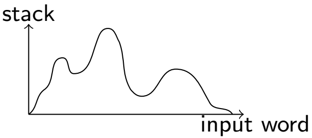
Determinization of visibly pushdown transducers



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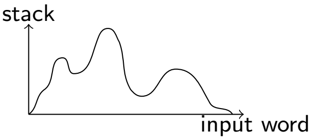

We did not succeed to solve this problem...
...but we found interesting open problems on word transducers

From Languages to Transductions

Languages	Transductions
Input $\rightarrow \{0,1\}$	*Input* $\rightarrow$ *Outputs*
automata	transducers
accept inputs	transform inputs

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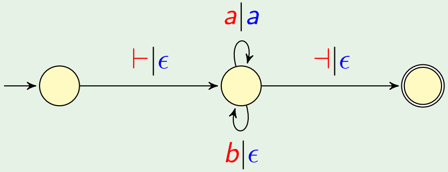
Applications:

- Word-to-word transducers:
  - language and speech processing
  - model-checking infinite state-space systems
  - verification of web sanitizers
  - string pattern matching
- Nested-word-to-word transducers:
  - XML transformations
  - model for recursive programs

(Two-way) finite state transducers

$=$ associate output words with transitions of a finite state automaton

Example (A transducer T)



Semantics $[\![T]\!]$: $f : \vdash w \dashv \; \mapsto a^{\#_a(w)}$, with $w \in \{a, b\}^*$

Non-determinism: semantics is a relation

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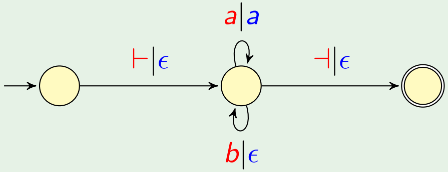

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A transducer is:

- functional if it realizes a function
- deterministic if the underlying automaton is deterministic

Classes: DFT, fNFT, NFT

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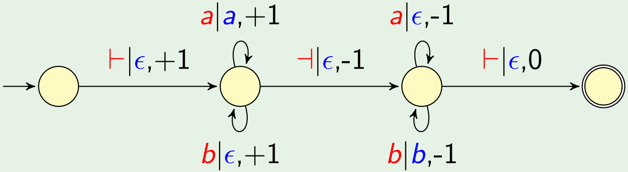

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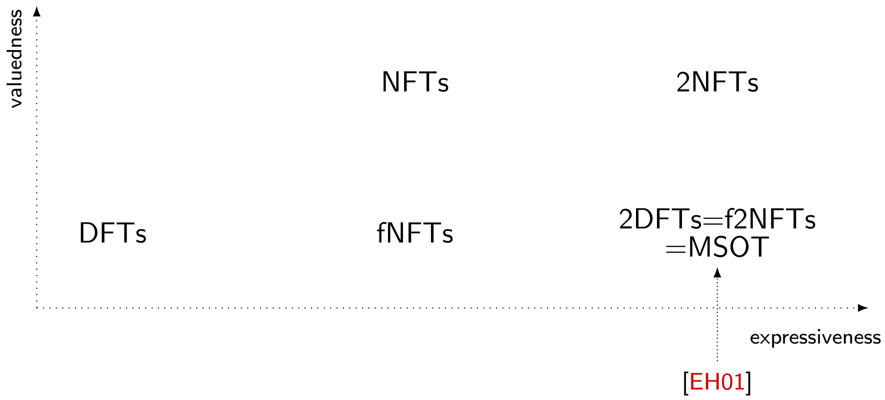
Classes: DFT, fNFT, NFT, 2DFT, f2NFT, 2NFT

Classes of Transductions



Classes of Transductions

$u \mapsto \mathit{mirror}(u)$

valuedness

NFTs $\subset$ 2NFTs

DFTs fNFTs $\subset$ 2DFTs=f2NFTs =MSOT

expressiveness

Classes of Transductions

valuedness

$\{(a, a), (a, b)\}$

NFTs $\subset$ 2NFTs

$\cup$ $\cup$

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expressiveness

Classes of Transductions

valuedness

$$
\begin{array}{ccccccc}
 & & u \mapsto \mathit{last}(u)^{|u|} & & \text{NFTs} & \subset & \text{2NFTs} \\
 & & & & \cup & & \cup \\
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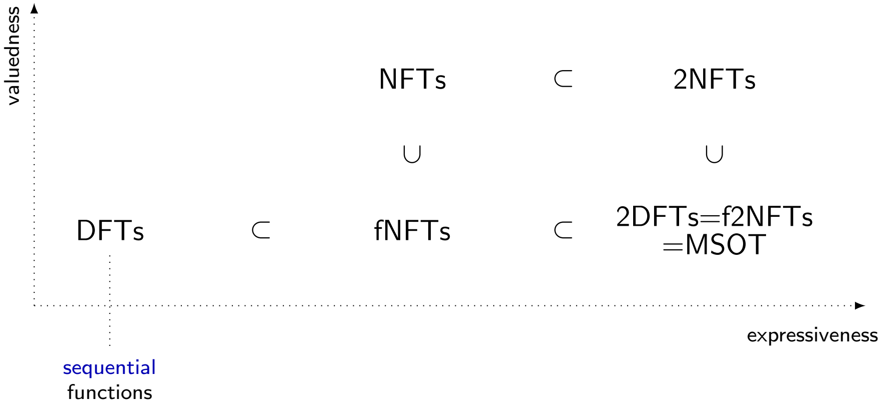

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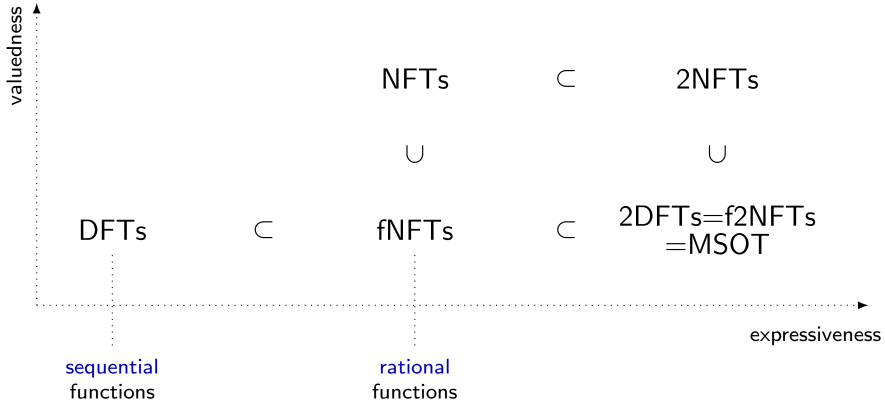

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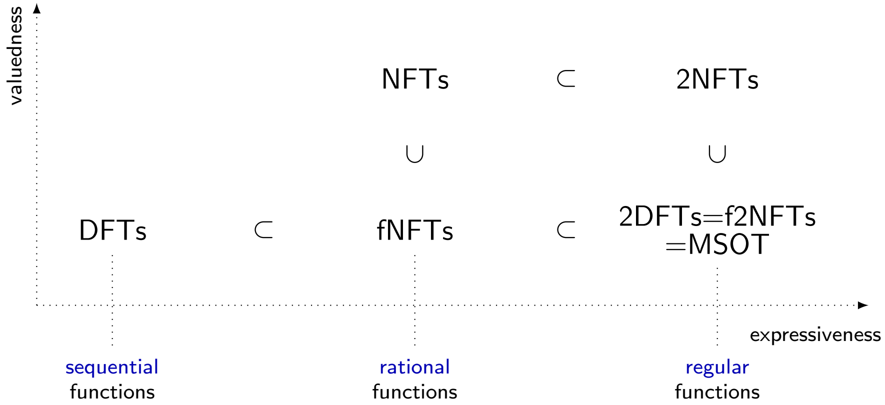
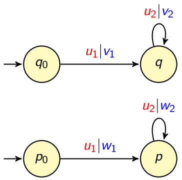


Classes of Transductions

Twinning Property



$\mathsf{delay}(v_1, w_1) = \mathsf{delay}(v_1 v_2, w_1 w_2)$

valuedness

NFTs $\subset$ 2NFTs

$\cup$ $\cup$

DFTs $\subset$ fNFTs $\subset$ 2DFTs=f2NFTs =MSOT

PTIME

[Choffrut77,WK95,BCPS03]

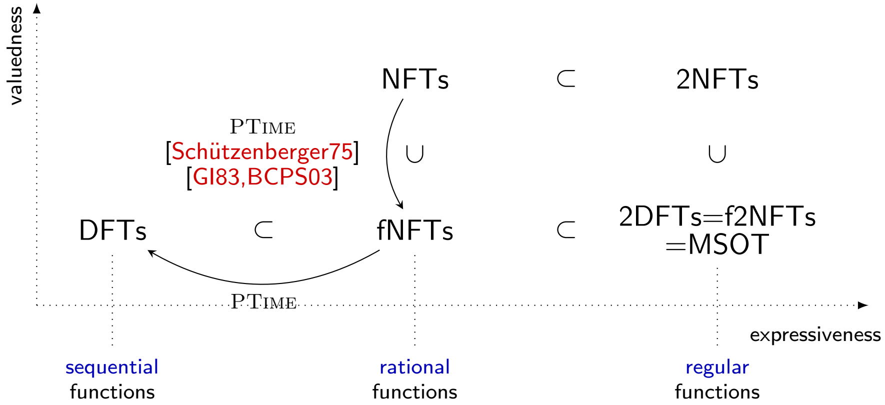
expressiveness

sequential functions

rational functions

regular functions

Classes of Transductions



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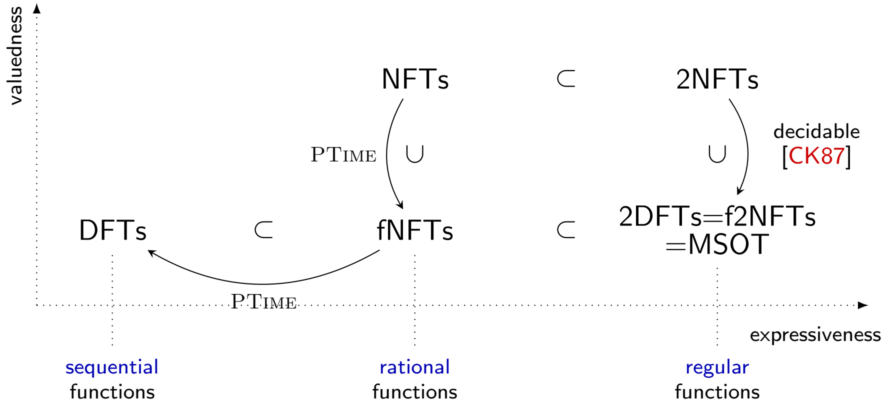

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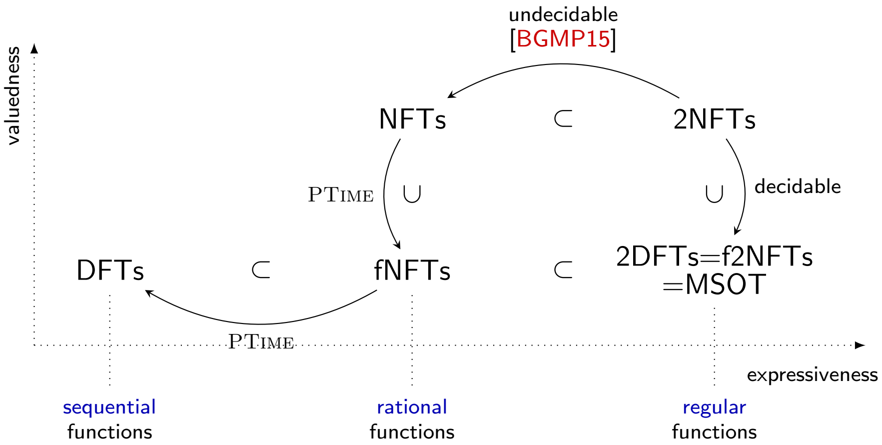

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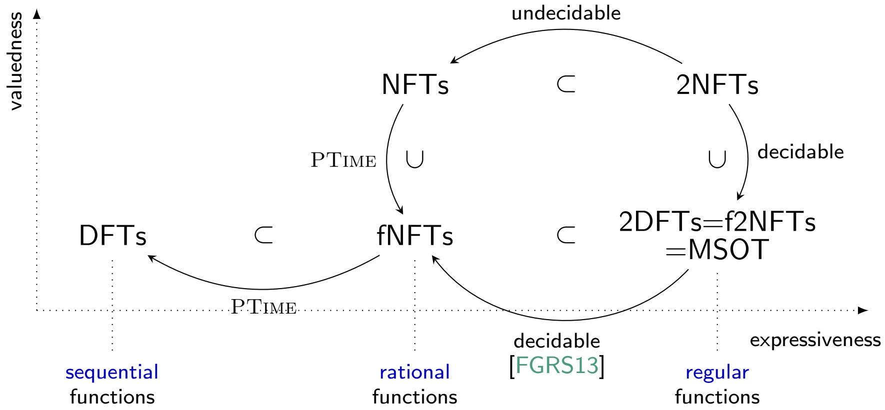

Simplification of models

Given a (complex) model of a transformation,
does there exist an equivalent simpler model?

➜ Natural question:

- minimization of automata

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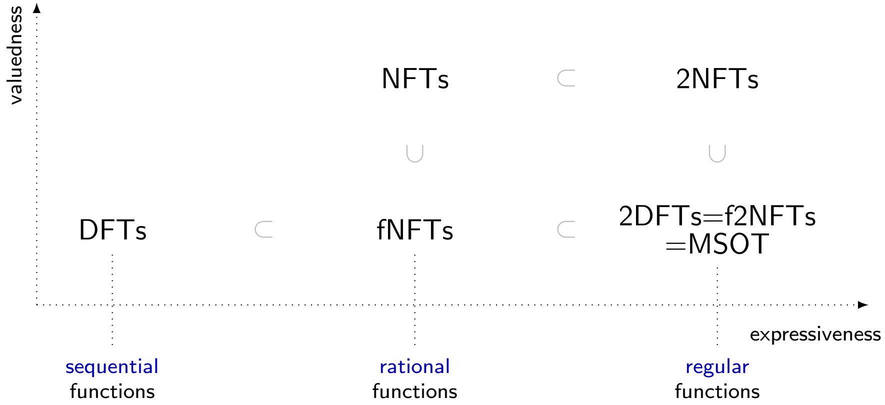
➜ Natural question:

- minimization of automata
- determinization
- 2way: minimize number of passes
- automata with registers: minimize number of registers
- ...

This talk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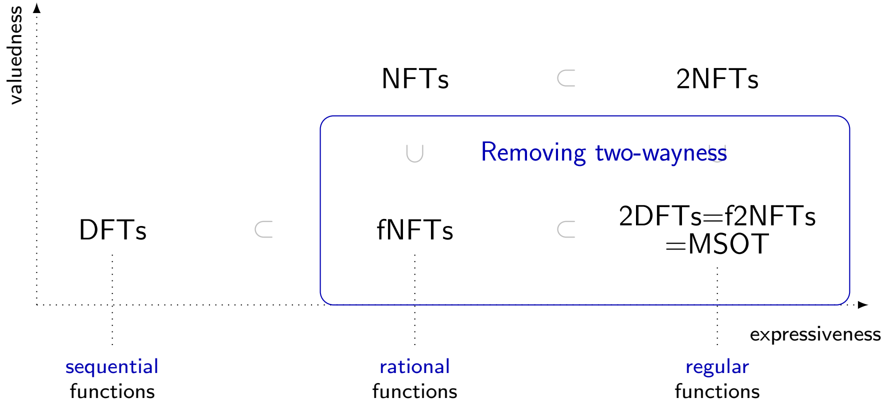
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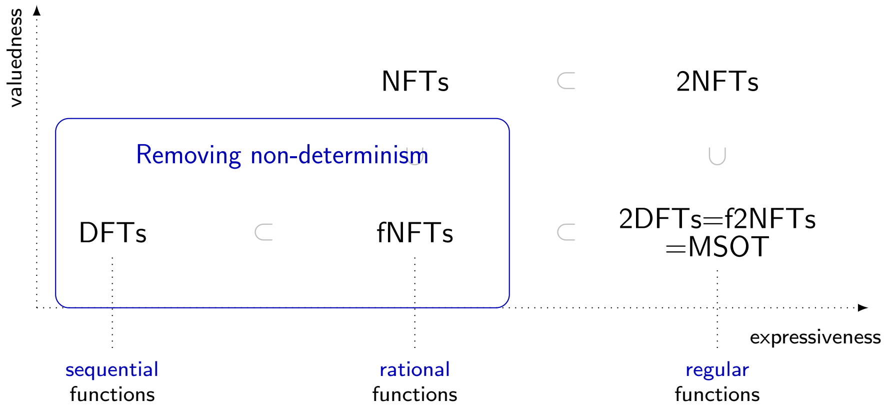
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Overview

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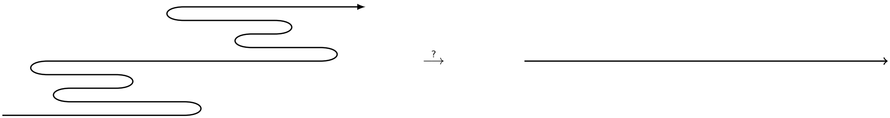
Rabin and Scott for transducers



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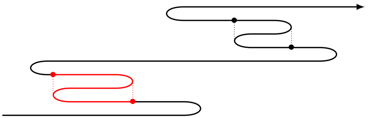

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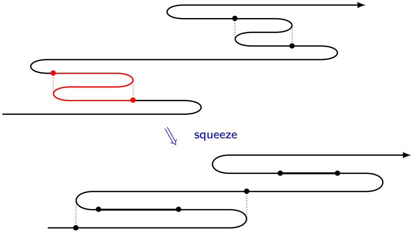

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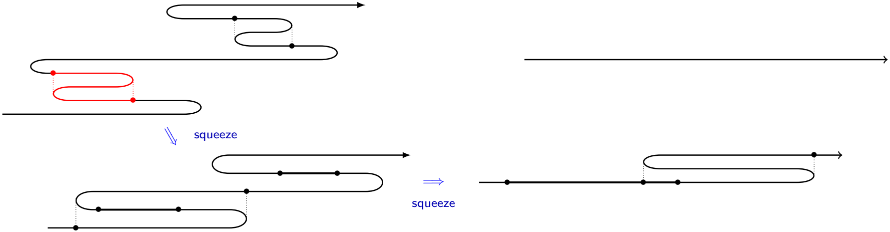

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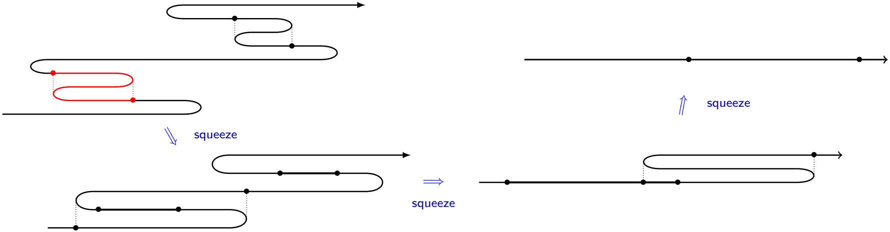

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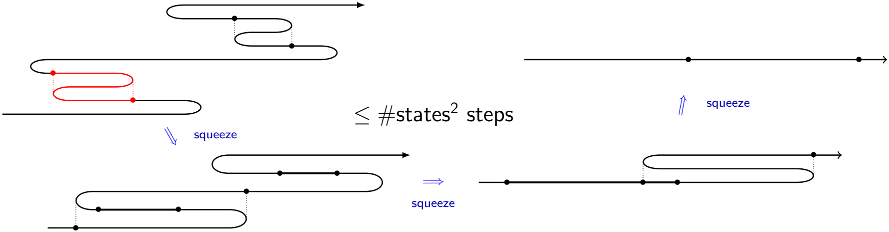

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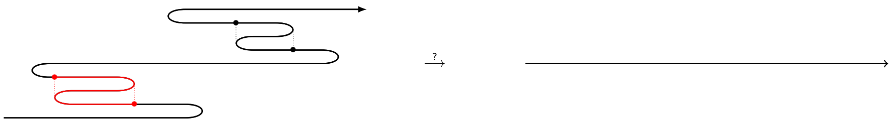

Lemma

If T is one-way definable then every zigzag transducer of T is also one-way definable.

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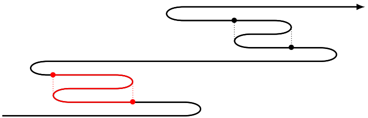

$\xrightarrow{?}$

Lemma

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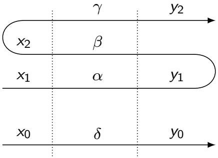
Decision Procedure:

1. repeat N^2 times:
   - are all zigzag transducers of T NFT-definable?
     - yes: $T \leftarrow \mathit{squeeze}(T)$
     - no: STOP: the initial 2NFTwas not NFT-definable!
2. remove backward transitions: you get an equivalent NFT

Rabin and Scott for transducers (2)

Consider a zigzag transducer Z



$$x_1 \cdot \alpha^n \cdot y_1 \cdot \beta^n \cdot x_2 \cdot \gamma^n \cdot y_2 = x_0 \cdot \delta^n \cdot y_0$$

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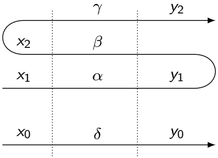

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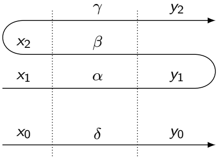

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$\Longrightarrow$ $\alpha,\beta,\gamma,\delta$ have conjugate primitive roots of length polynomial in $|Z|$

Theorem

Given a zigzag transducer Z, we build a fNFT Z' s.t.:

1. $|Z'| = exp(O(|Z|))$
2. $R(Z') \subseteq R(Z)$
3. *Z is NFT-definable $\iff$ $dom(Z) \subseteq dom(Z')$*

3. is decidable.

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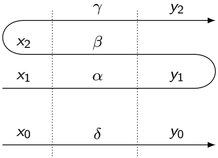

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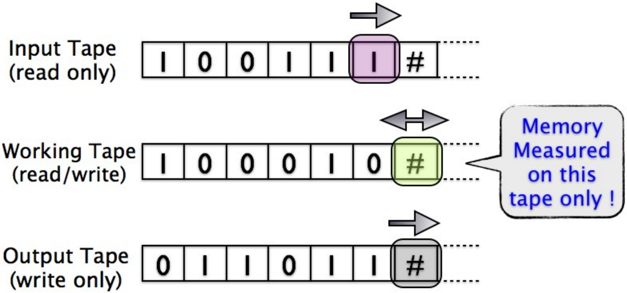
Yields a non-elementary decision procedure

Streaming model: Deterministic Turing Transducer



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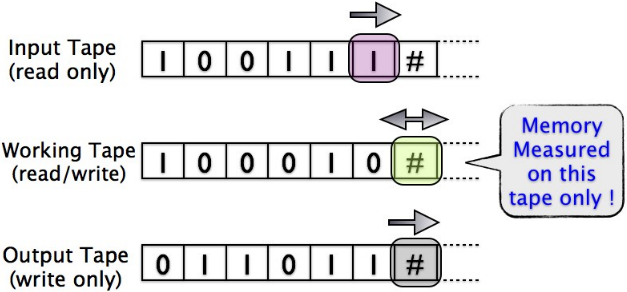

Bounded Memory Problem

Input: a transformation T

Output: can T be realized with bounded memory?

$$\exists B \in \mathbb{N} \cdot \forall u \in dom(T)$$
$$T(u) \text{ can be computed with } B\text{-bounded memory?}$$

Streamability of word-to-word transformations

Observation:

$f : \Sigma^* \to \Sigma^*$ is bounded memory iff it is DFT-definable.

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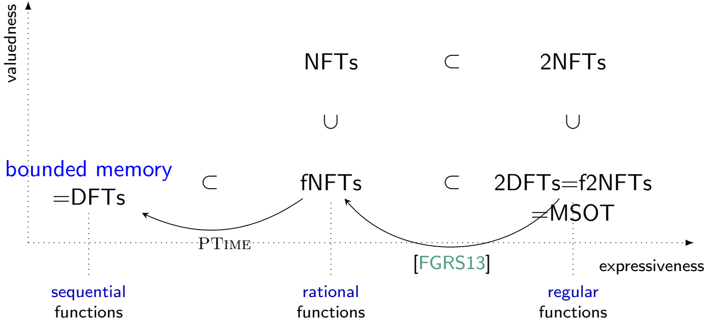

Corollary

Bounded memory is decidable for regular word functions.

Recent works on removing two-wayness

Two new techniques to obtain elementary complexity:

- [BGMP17] (Bordeaux)
  - careful analysis of loops
  - algebraic techniques (idempotent element, Simon)
  - decision in 2ExpSpace
  - also yields decidability results related to sweeping transducers

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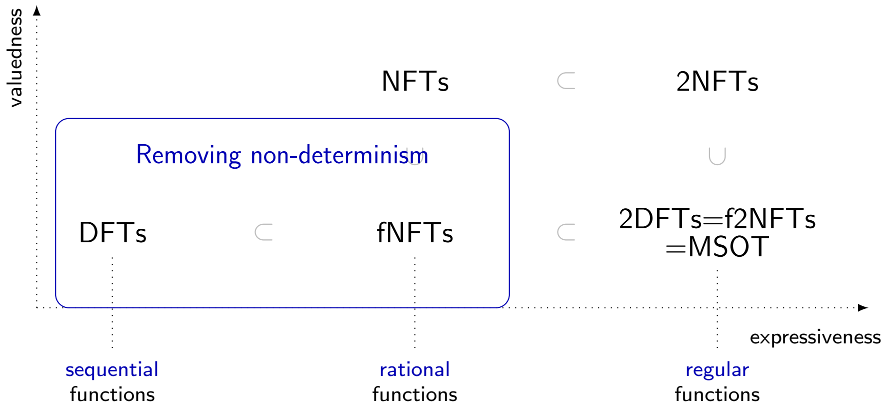
- [MRT17] (Marseille, unpublished)
  - normalization procedure
  - more amenable to implementation
  - promising technique to address determinization of visibly pushdown transducers

Overview

Removing non-determinism



Multi-sequential functions

Fact: Not every one-way transducer can be determinized

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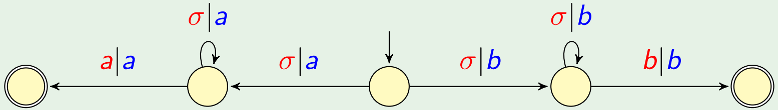

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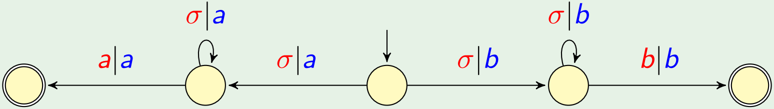

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But $\textsc{Last}$ can be written as the "sum" of two sequential functions

Multi-sequential functions

Fact: Not every one-way transducer can be determinized

Definition ([CS86])

Multi-sequential functions are defined as functions that can be realized as finite union of sequential transducers.

Motivation: parallel evaluation of the function

Examples: ($\textsc{Last}$: $w\sigma \mapsto \sigma^{|w|+1}$, with $\sigma \in \{a, b\}, w \in \{a, b\}^+$)

- $\textsc{Last}$ is multi-sequential: split the domain as $\Sigma^* a \uplus \Sigma^* b$

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- $\textsc{Last}^* : u_1 \# \ldots \# u_n \mapsto \textsc{Last}(u_1) \# \ldots \# \textsc{Last}(u_n)$ is not multi-seq.

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Theorem ([CS86,FJ15])

Multi-sequentiality can be decided in Ptime.

This work: minimization of the size of the union.

From rational to sequential functions [Choffrut77]

Sequentiality Problem

Input: a fNFT

Question: does there exist an equivalent DFT?

Theorem ([Choffrut77])

Let T be a fNFT. T.f.a.e:

- *there exists a DFT T' s.t. $T \equiv T'$*
- *T satisfies the twinning property*
- *$[\![T]\!]$ satisfies the Lipschitz property*
  $\exists L \in \mathbb{N} \mid \forall u, v \in dom(f), d(f(u), f(v)) \leq L.d(u, v)$

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Twinning Property can be decided in PTime ([WK95])

Corollary

The Sequentiality Problem is decidable in PTime.

Twinning Property [Choffrut77]

Define:

$$\mathrm{delay}(u, v) = \mathrm{lcp}(u, v)^{-1}.(u, v)$$

Example:
$\mathrm{lcp}(aaa, aab) = aa$
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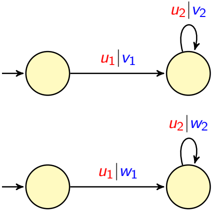

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Twinning Property [Choffrut77]

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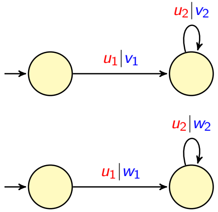

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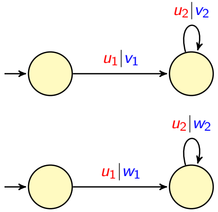

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Lemma

$T \models$ Twinning Property
$\Longrightarrow$ *bounded delays*
$\Longrightarrow$ *termination of subset constr.*

Characterization of k-sequential functions

k-sequentiality Problem

Input: a fNFT T and $k \in \mathbb{N}$

Question: does there exist DFTs $T_1, \ldots, T_k$ s.t. $T \equiv \bigcup_i T_i$?

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The k-sequentiality Problem is PSpace-complete (k given in unary).

Branching twinning property of order k

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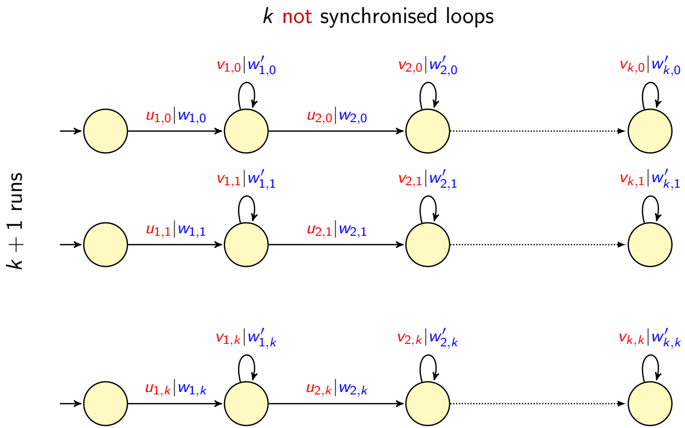
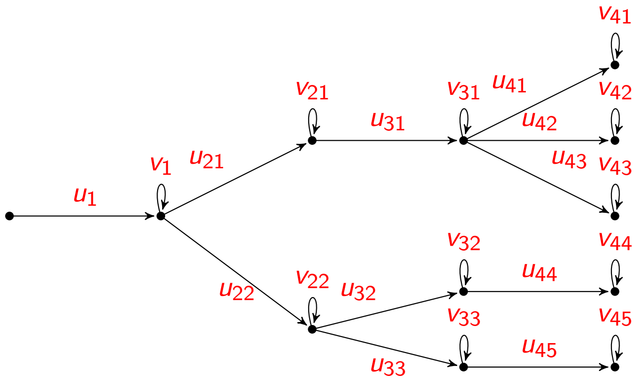


there are two runs $0 \leq i < j \leq k$ s.t. for every loop ℓ with same input words,

we have $\text{delay}(w_{1,i} \dots w_{\ell,i}, w_{1,j} \dots w_{\ell,j}) = \text{delay}(w_{1,i} \dots w_{\ell,i} w'_{\ell,i}, w_{1,j} \dots w_{\ell,j} w'_{\ell,j})$

Branching twinning property of order k

Tree representation of input words:



Branching twinning property of order k

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A fNFT is definable by a union of k DFT iff it satisfies the BTP of order k.

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Induction on k; case $k = 1$ already solved

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Consider subset construction with delays

- if a delay is too large in some set S, find a loop
- split S into $S' \uplus S''$, depending on what happens on the loop
- define k' as the smallest integer s.t. $T_{S'} \models BTP_{k'}$ (same with k'')
- $T \models BTP_k$ entails $k' + k'' \leq k$
- apply induction hypothesis

Overview

Summary

- Streamability/efficient evaluation of transformations
- Open problems on word transducers
- Removing two-wayness: decidable, now in 2ExpSpace
- Removing non-determinism: k-sequentiality PSpace-complete

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- logical presentation of transductions
- functional $\rightsquigarrow$ finite-valued
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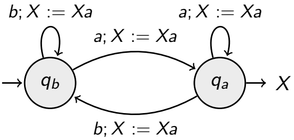
Growing interest for transducers:

- ~~LIF~~ LIS
- LaBRI (2Way to 1Way)
- LSV!
- Warsaw (origin semantics)
- Microsoft (symbolic transducers)
- UPenn (streaming string transducers)

Streaming String Transducers

Definition ([AC10])

Streaming String Transducer = DFA
+ finite set of registers X, Y, Z



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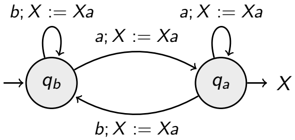

➜ allows to recover previously defined classes of functions:

k-sequential functions

k registers $X_1, \ldots, X_k$
$X_i := X_i u$

Rational functions

$X := Yu$

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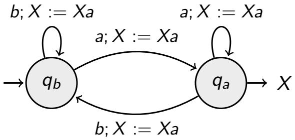

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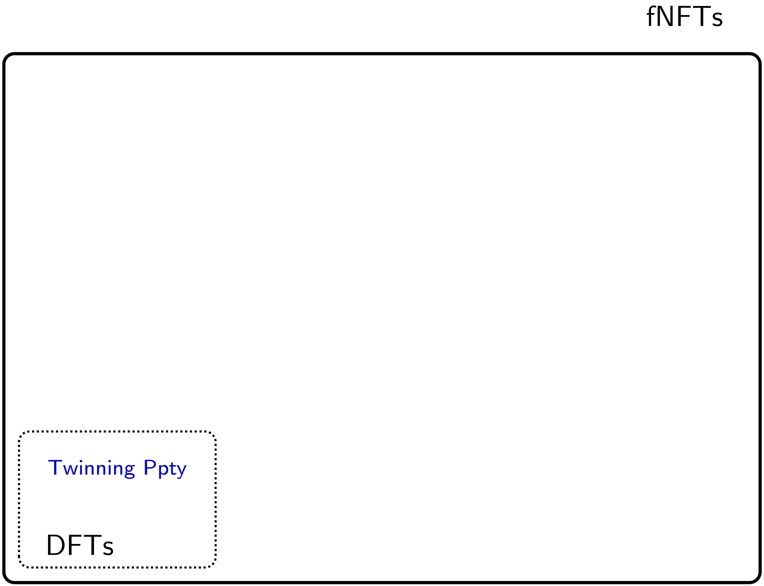
Theorem

One can minimize the number of registers in the class of rational functions.

Landscape of rational functions



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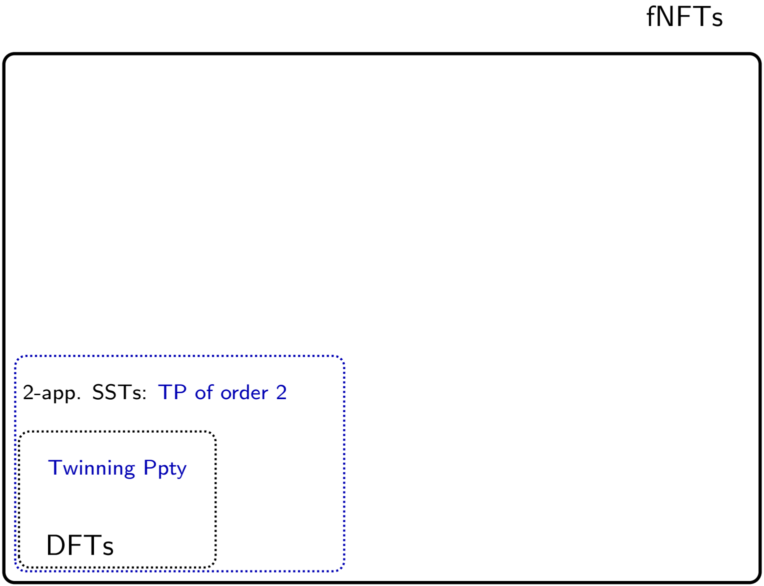

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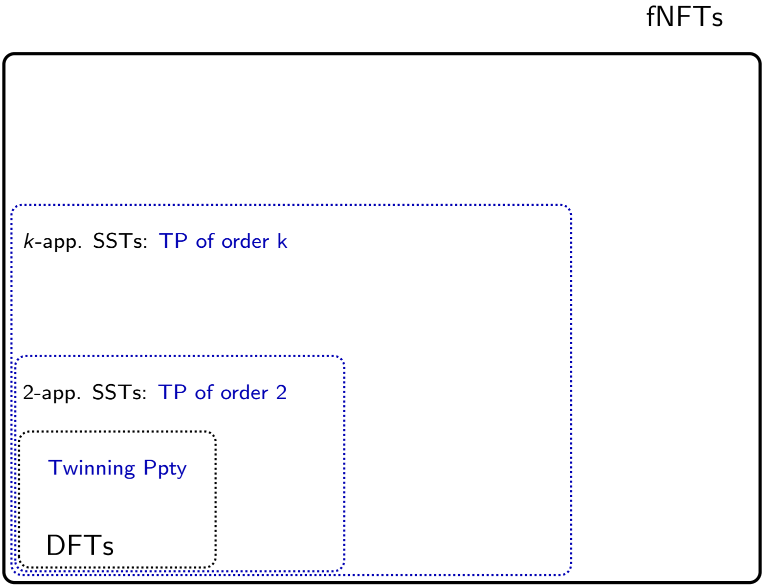

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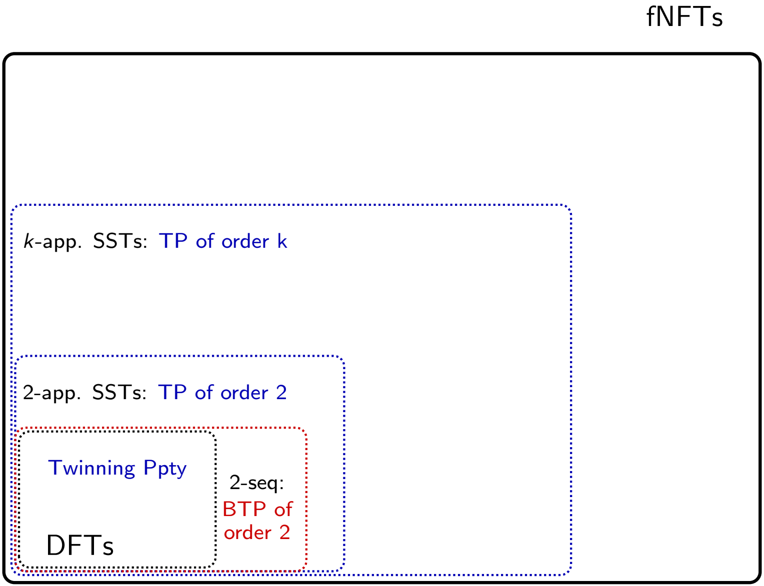

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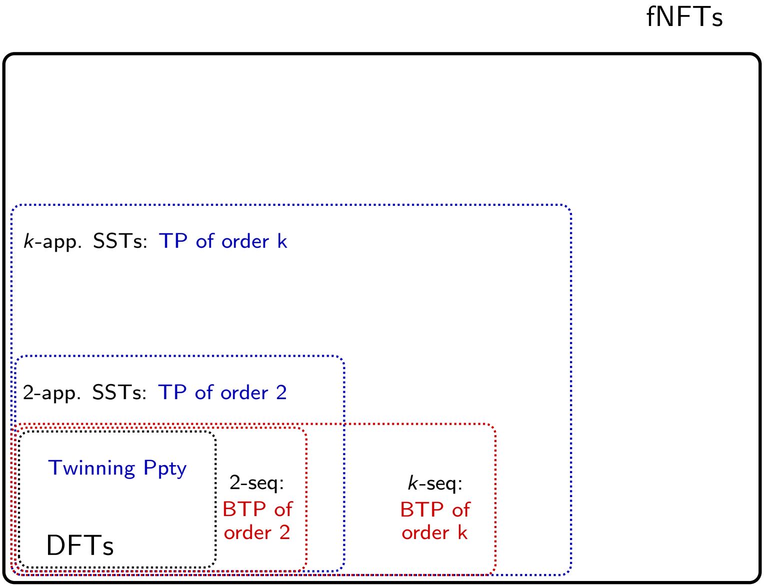

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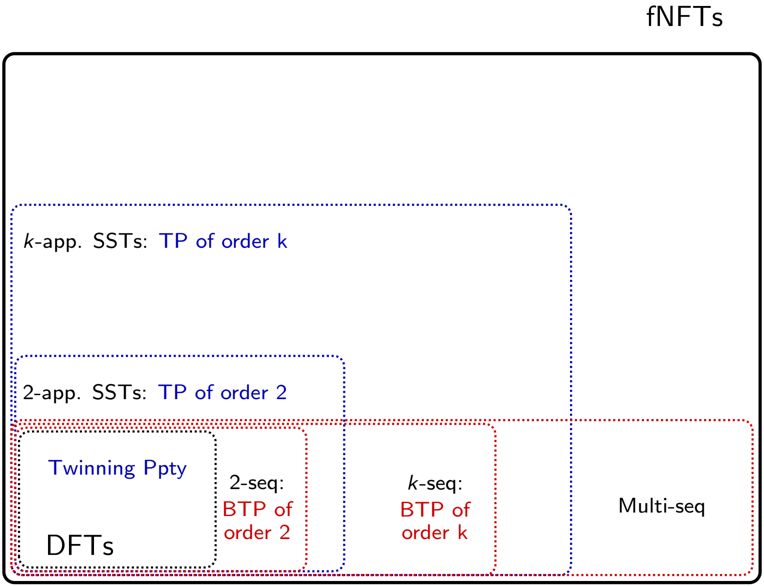

Some perspectives

- Register minimization for regular functions
  ➜ deal with prepending of words ($X := uYv$)
  ➜ deal with concatenation of registers ($X := YZ$)
- Algebraic characterizations (bimachines [RS91])
- Specification formalism
- Tool development
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