Verification of Population Protocols

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Joint work with
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and
Michael Blondin, Stefan Jaax, and Philipp Meyer

Deaf Black Ninjas in the Dark

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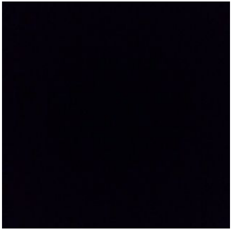

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- Deaf Black Ninjas meet at a Zen garden in the dark
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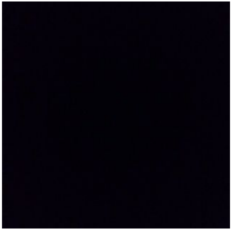
- How can they conduct the vote?



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- Initially all Ninjas are Active, and their initial estimation is their own vote
- Ninjas follow this protocol:

(YA , NA)	$\rightarrow$	(NP , NP)	(opposite votes "cancel")
(YA , NP)	$\rightarrow$	(YA , YP)	(active "survivors" tell
(NA , YP)	$\rightarrow$	(NA , NP)	outcome to passive Ninjas)
(NP , YP)	$\rightarrow$	(NP , NP)	(to deal with ties)

Population protocols (PP)

Theoretical model for distributed computation

Proposed in 2004 by Angluin *et al.*

Designed to model collections of

identical, finite-state, and mobile agents

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Syntax

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Intuition:

if	$(q_1, q_2) \mapsto (q'_1, q'_2) \in \Delta$ **and**
	two agents in states q_1 and q_2 "meet",
then	the agents can interact and
	change their states to q'_1, q'_2.

Assumption: at least one interaction for each pair of states (possibly $(q_1, q_2) \mapsto (q_1, q_2)$)

Semantics

Configuration: mapping $C\colon Q \to \mathbb{N}$, where $C(q)$ is the current number of agents in state q.

q_1	q_2	q_3	q_4
2	1	0	3

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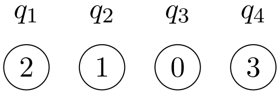

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$$\begin{array}{cccc} q_1 & q_2 & q_3 & q_4 \\ 2 & 1 & 0 & 3 \end{array} \longrightarrow \begin{array}{cccc} q_1 & q_2 & q_3 & q_4 \\ 1 & 0 & 1 & 4 \end{array}$$

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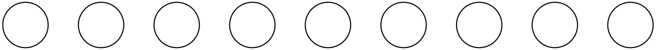
Execution: infinite sequence $C_0 \to C_1 \to C_2 \to \cdots$ of steps

Population protocols (PPs)

A population protocol (PP) consists of

- A PP-scheme (Q, Δ)

Q:



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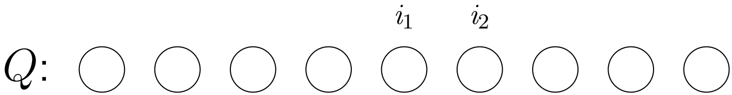

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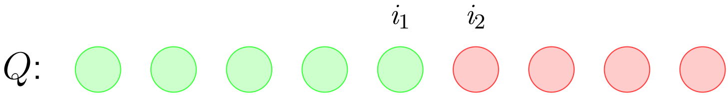
- A partition of Q into 1-states (green) and 0-states (pink)



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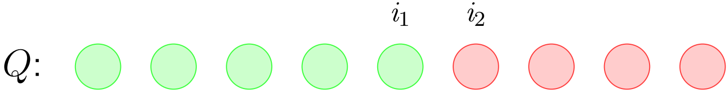

An execution reaches consensus $b \in \{0, 1\}$ if from some point on every agent stays within the b-states.

Computing with PPs

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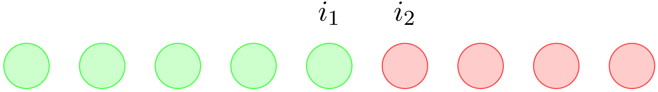

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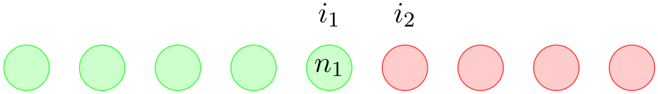

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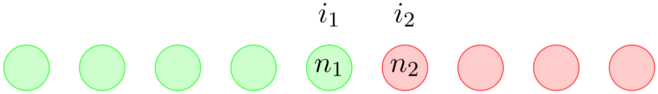

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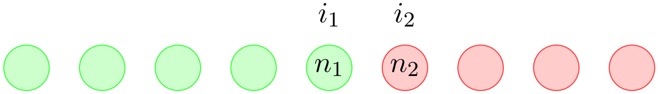

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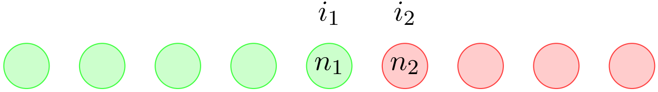

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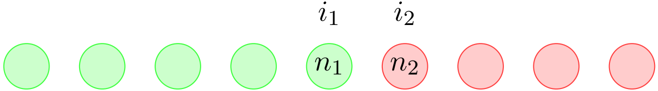

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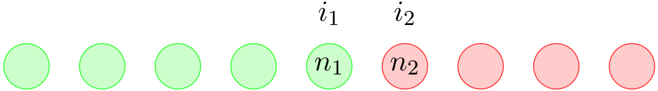

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A PP computes $P(x_1, \ldots, x_n) \colon \mathbb{N}^n \rightarrow \{0, 1\}$ if it computes $P(n_1, \ldots, n_k)$ for every input $(n_1, \ldots, n_k)$

Previous work

Expressive power thoroughly studied:

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- PPs compute exactly the Presburger predicates (Angluin *et al.* 2007)
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- Private computation in PPs (Delporte-Gallet *et al.* 2007)
- PPs with identifiers (Guerraoui *et al.* 2007)
- PPs with a leader (Angluin *et al.* 2008)
- Mediated PPs (Michail *et al.*, 2011)
- Trustful PPs (Bournez *et al.*, 2013)

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Well-specification problem: Given a protocol, decide if it is well-specified.

Correctness problem: Given a protocol and a Presburger predicate, decide if the protocol is well-specified and computes the predicate.

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Not complete or not automatic.

Main results

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Theorem: The reachability problem for Petri nets can be reduced to the well-specification and correctness problems for PPs.

From PPs to Petri nets

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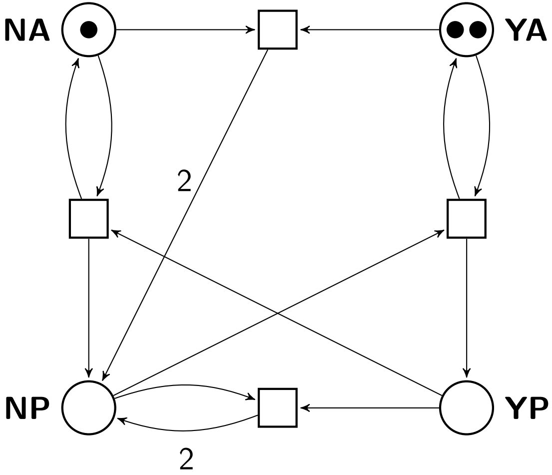
Configuration graph	Reachability graph
PP	Net $+$ infinite family of initial markings

Petri net of the majority protocol



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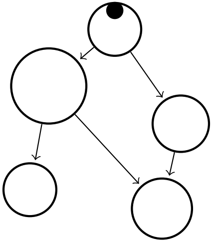
Fact: Every execution of a PP gets eventually trapped in a bottom SCC of its configuration graph w.p.1, and visits all configurations of the SCC infinitely often w.p.1.



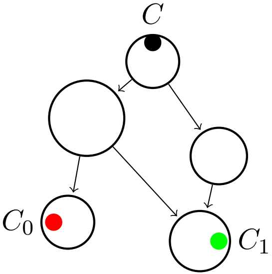
Reducing well-specification to a reachability problem

Bottom configuration: configuration of a bottom SCC.

Fact: A PP is ill-specified iff there is

- an initial configuration C, and
- two bottom configurations C_0 and C_1, reachable from C

such that C_0 has at least one agent in a 0-state, and C_1 has at least one agent in a 1-state.



Well-specification is decidable

Theorem 1: Given two (possibly infinite!) Presburger sets of configurations $\mathcal{C}_1$, $\mathcal{C}_2$ of a Petri net, it is decidable if some configuration of $\mathcal{C}_2$ is reachable from some configuration of $\mathcal{C}_1$.

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Effectiveness follows from a bound on the size of the Presburger representation due to Leroux (2011).

Well-specification is decidable

- $\mathcal{S}$: protocol scheme
- $\mathcal{I}$: set of all initial configurations
- $\mathcal{B}_b$ (where $b \in \{0, 1\}$): set of all bottom configurations with at least one agent in a b-state

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- Check if $\mathcal{B}_0 \times \mathcal{B}_1$ is reachable from $\mathcal{I}_2$
  $\mathcal{B}_0 \times \mathcal{B}_1$ is effectively Presburger by Theorem 2, the check is effective by Theorem 1.

More decidable problems

Given: A PP P, a Presburger predicate Π
Decide: Does P compute Π?

Given: A well-specified PP P
Compute: A Presburger formula (or semilinear representation of) the predicate computed by P.

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Equivalent definition: bottom SCCs contain only one configuration.

Proposition: WS^2 protocols (well specified and silent) can compute all Presburger predicates.

Fighting complexity I: The class WS^2

Search for a subclass of the class WS of well-specified protocols that

- Has a membership problem of reasonable complexity.
- Can compute all Presburger predicates.

Many protocols from the literature are silent: Executions end w.p.1 in terminal configurations that enable no transitions.

Equivalent definition: bottom SCCs contain only one configuration.

Proposition: WS^2 protocols (well specified and silent) can compute all Presburger predicates.

Proposition : Petri net reachability is reducible to the membership problem for WS^2.

Fighting complexity II: The class WS^3

WS^2: Well-sp. silent

Termination

For every reachable configuration C there exists an execution leading from C to a terminal conf. $C_\perp$

Consensus

All terminal configurations reachable from a given initial configuration form the same consensus.

Fighting complexity II: The class WS^3

WS^2: Well-sp. silent

Termination

For every reachable configuration C there exists an execution leading from C to a terminal conf. $C_\perp$

Consensus

All terminal configurations reachable from a given initial configuration form the same consensus.

WS^3: Well-sp. strongly silent

Layered Termination

For every configuration C there exists a layered execution leading from C to a terminal configuration $C_\perp$

Strong Consensus

All terminal configurations potentially reachable from a given initial configuration form the same consensus.

Layered Termination

A protocol is layered if there is a partition of the set T of transitions into layers $T_1, \dots T_n$ s.t. for every configuration C (reachable or not):

- all executions from C containing only transitions of a single layer are finite.
- if all transitions of T_i are disabled at C, then they cannot be re-enabled by any sequence of transitions of $T_{i+1}, \dots, T_n$.

An execution is layered if it "respects the layers", i.e., if it belongs to $T_1^* T_2^* \dots T_n^*$.

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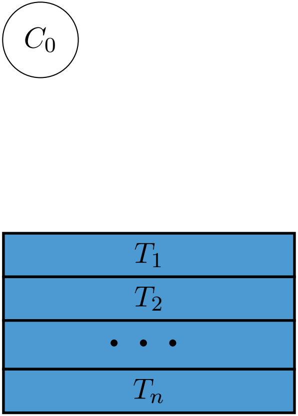
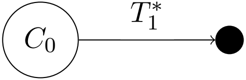
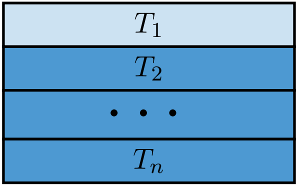
Fact: For every configuration C (reachable or not) there exists a layered execution leading from C to a terminal configuration $C_\perp$.

Layered Termination



Layered Termination





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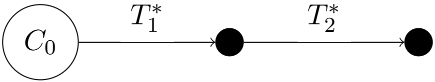

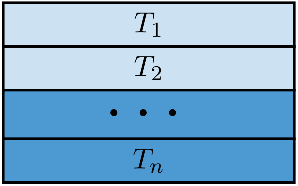

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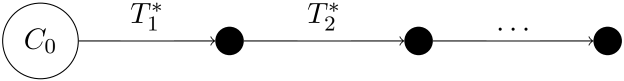

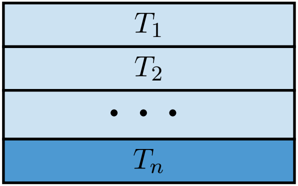

Layered Termination

$$C_0 \xrightarrow{T_1^*} \bullet \xrightarrow{T_2^*} \bullet \xrightarrow{\ \dots\ } \bullet \xrightarrow{T_n^*} C_\perp$$

T_1
T_2
$\bullet\ \bullet\ \bullet$
T_n

Complexity of checking Layered Termination

Lemma: Deciding Layered Termination is in NP.

Complexity of checking Layered Termination

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Proof sketch:

- Guess layers.
- Test that each individual layer terminates.
  Reducible to a Linear Programming Problem
- Test that lower layers cannot re-enable higher layers.
  Simple syntactic check.

Strong Consensus

Replace reachability by *cruder* relation called *potential reachability*:

$$\begin{aligned} \text{Reachability} &\Longrightarrow \text{Potential Reachability} \\ \text{Potential Reachability} &\not\Longrightarrow \text{Reachability} \end{aligned}$$

Potential reachability is defined in terms of a class of linear invariants derivable from the Petri net of the protocol by syntactic means (place invariants, siphons, traps).

A configuration C' is potentially reachable from C if both C and C' satisfy the same invariants of the class.

Lemma: Deciding Strong Consensus is in co-NP.

Completeness

Lemma: All well-specified population protocols can be represented by an equivalent population protocol satisfying Layered Termination and Strong Consensus.

By quantifier elimination, all predicates computable by Population Protocols can be defined as boolean combinations of

- Threshold: Is the weighted sum of the input values larger than a given threshold?

$$\Sigma_i \alpha_i x_i > c$$

- Remainder: Is the sum of the input values modulo a given m equal to a given c?

$$\Sigma_i \alpha_i x_i \bmod m = c$$

- Give WS^3 protocols for Threshold and Remainder predicates
- Prove that WS^3 protocols are closed under conjunction and negation.

Implementation

On top of the SMT-solver **Z3**.

Our tool reads a protocol and constructs two sets of constraints:

- The first is satisfiable iff. Layered Termination holds.
- The second is unsatisfiable iff. Strong Consensus holds.

Protocols from the literature for Majority, Threshold, Remainder, etc. belong to WS^3.

Experimental Results

Experiments were performed on a machine equipped with an Intel Core i7-4810MQ CPU and 16 GB of RAM.

Threshold			
ℓ_{max}	$\lvert Q\rvert$	$\lvert T\rvert$	Time[s]
3	28	288	8.0
4	36	478	26.5
5	44	716	97.6
6	52	1002	243.4
7	60	1336	565.0
8	68	1718	1019.7
9	76	2148	2375.9
10	84	2626	timeout

Remainder			
m	$\lvert Q\rvert$	$\lvert T\rvert$	Time[s]
10	12	65	0.4
20	22	230	2.8
30	32	495	15.9
40	42	860	79.3
50	52	1325	440.3
60	62	1890	3055.4
70	72	2555	3176.5
80	82	3320	timeout

Experimental Results

Flock of birds[1]			
c	$\|Q\|$	$\|T\|$	Time[s]
20	21	210	1.5
25	26	325	3.3
30	31	465	7.7
35	36	630	20.8
40	41	820	106.9
45	46	1035	295.6
50	51	1275	181.6
55	56	1540	timeout

[1] Chatzigiannakis et al., 2010

Flock of birds[2]			
c	$\|Q\|$	$\|T\|$	Time[s]
50	51	99	11.8
100	101	199	44.8
150	151	299	369.1
200	201	399	778.8
250	251	499	1554.2
300	301	599	2782.5
325	326	649	3470.8
350	351	699	timeout

[2] Clement et al., 2011

Conclusions

- The natural verification problems for population protocols are decidable.
- Efficient verification algorithms for the class WS^3.
- Implementation on top of SMT-solvers.

Conclusions

- The natural verification problems for population protocols are decidable.
- Efficient verification algorithms for the class WS^3.
- Implementation on top of SMT-solvers.
- Many open questions:
  - Complexity for immediate observation and immediate transmission protocols
  - Continuous Petri nets as abstractions
  - Expressive power of PP in non-uniform computational models
  - Applications to theoretical chemistry and systems biology
  - Correctness problem and convergence speed for WS^3 protocols.
  - Fault localization and repair.
  - Synthesis of WS^3 protocols.

Thank You