TD 6

Exercise 1 (Ultimately Periodic Words)**.** An *ultimately periodic word* over Σ is a word of form $u \cdot v^\omega$ with u in Σ^* and v in Σ^+.

Prove that any nonempty recognizable language in $\mathsf{Rec}(\Sigma^\omega)$ contains an ultimately periodic word.

Exercise 2 (Muller Automata)**.** A nondeterministic Muller automaton is a tuple $\mathcal{A} = (Q, \Sigma, I, T, \mathcal{F})$, where Q, Σ, I, T are as for Büchi automata and $\mathcal{F} \subseteq 2^Q$ is the acceptance condition. For a run σ of $\mathcal{A}$, denote by $\mathsf{Inf}(\sigma)$ the set of states which are visited infinitely often. A run σ is accepting if $\mathsf{Inf}(\sigma) \in \mathcal{F}$.

1. Give a *deterministic* Muller automaton for the language $(a + b)^* a^\omega$.
2. Show that for any Muller automaton $\mathcal{A}$, $L(\mathcal{A})$ is ω-regular.
3. Show that any ω-regular language is accepted by some (nondeterministic) Muller automaton.

Remark: in fact, any ω-regular language can be recognized by some *deterministic* Muller automaton.

Exercise 3 (Synchronous Büchi Transducers)**.** Give unambigous synchronous Büchi transducers for the following formulæ:

1. $\mathsf{SF}\, q$
2. $\mathsf{SG}\, q$
3. $\mathsf{G}(p \to \mathsf{F}\, q)$