Exercise 1 (Equivalences). Are the following formulæ equivalent?

1. $A X A G \varphi$ and $A X G \varphi$
2. $E X E G \varphi$ and $E X G \varphi$
3. $A (\varphi \land \psi)$ and $A \varphi \land A \psi$
4. $E (\varphi \land \psi)$ and $E \varphi \land E \psi$
5. $\neg A (\varphi \Rightarrow \psi)$ and $E (\varphi \land \neg \psi)$

Exercise 2 (Semantics of CTL*). Compute $\llbracket \varphi \rrbracket$, where:

$\varphi = A [(Xq) \lor FA((EFG)p)(UAGq)]$

Exercise 3 (CTL Model-Checking). Let $M = (S, T, I, AP, \ell)$ be a finite Kripke structure, and $\varphi$ a CTL formula.

1. Let $M_{\varphi}$ be the restriction of $M$ to states satisfying $\varphi$: $M_{\varphi} = (\llbracket \varphi \rrbracket, T \cap \llbracket \varphi \rrbracket, I \cap \llbracket \varphi \rrbracket, AP, \ell|_{\llbracket \varphi \rrbracket})$.
   Show that $s \in \llbracket EG \varphi \rrbracket$ iff there exists a non-trivial strongly connected component $C$ of $M_{\varphi}$ and $t \in C$ such that $s \rightarrow^* t$ in $M_{\varphi}$.
2. Deduce an algorithm to compute $\llbracket EG \varphi \rrbracket$ from $M$ and $\llbracket \varphi \rrbracket$. What is the complexity of your procedure?

Exercise 4 (CTL^+). CTL^+ extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

\[
\begin{align*}
  f & ::= \top \mid a \mid f \land g \mid \neg f \mid E \varphi \mid A \varphi \\
  \varphi & ::= \varphi \land \psi \mid \neg \varphi \mid X f \mid f \lor g
\end{align*}
\]

where $a$ is an atomic proposition. The associated semantics is that of CTL*.

We want to prove that, for any CTL^+ formula, there exists an equivalent CTL formula.
1. Give an equivalent CTL formula for
\[ E((a_1 \mathbf{U} b_1) \land (a_2 \mathbf{U} b_2)) \].

2. Generalize your translation for any formula of form
\[ E \left( \bigwedge_{i=1,\ldots,n} (\psi_i \mathbf{U} \psi'_i) \land \mathbf{G} \varphi \right). \] (1)

What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL\(^+\) formula:
\[ E(Xa \land (b \mathbf{U} c)) \].

4. Using subformulae of form (1) and E modalities, give an equivalent CTL formula to
\[ E(X\varphi \land \bigwedge_{i=1,\ldots,n} (\psi_i \mathbf{U} \psi'_i) \land \mathbf{G} \varphi'). \] (2)

What is the complexity of your translation?

5. We only have to transform any CTL\(^+\) formula into (nested) disjuncts of form (2). Detail this translation for the following formula:
\[ A((F a \lor X a \lor X \neg b \lor F \neg d) \land (d \mathbf{U} \neg c)) \].

2