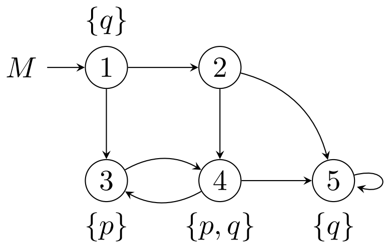
TD 3: CTL, CTL*

Exercise 1 (Equivalences)**.** Are the following formulæ equivalent?

1. $\mathsf{A}\,\mathsf{X}\,\mathsf{A}\,\mathsf{G}\,\varphi$ and $\mathsf{A}\,\mathsf{X}\,\mathsf{G}\,\varphi$
2. $\mathsf{E}\,\mathsf{X}\,\mathsf{E}\,\mathsf{G}\,\varphi$ and $\mathsf{E}\,\mathsf{X}\,\mathsf{G}\,\varphi$
3. $\mathsf{A}(\varphi \wedge \psi)$ and $\mathsf{A}\,\varphi \wedge \mathsf{A}\,\psi$
4. $\mathsf{E}(\varphi \wedge \psi)$ and $\mathsf{E}\,\varphi \wedge \mathsf{E}\,\psi$
5. $\neg\,\mathsf{A}(\varphi \Rightarrow \psi)$ and $\mathsf{E}(\varphi \wedge \neg\psi)$

Exercise 2 (Semantics of CTL*)**.** Compute $[\![\varphi]\!]$, where:



$$\varphi = \mathsf{A}\big[(\mathsf{X}q) \vee \mathsf{F}\mathsf{A}\big((\mathsf{E}\mathsf{F}\mathsf{G}p)\mathsf{U}(\mathsf{A}\mathsf{G}q)\big)\big]$$

Exercise 3 (CTL Model-Checking)**.** Let $M = (S, T, I, \mathrm{AP}, \ell)$ be a finite Kripke structure, and φ a CTL formula.

1. Let M_φ be the restriction of M to states satisfying φ: $M_\varphi = ([\![\varphi]\!], T \cap [\![\varphi]\!]^2, I \cap [\![\varphi]\!], \mathrm{AP}, \ell_{|[\![\varphi]\!]})$.
   Show that $s \in [\![\mathsf{E}\mathsf{G}\,\varphi]\!]$ iff there exists a non-trivial strongly connected component C of M_φ and $t \in C$ such that $s \to^* t$ in M_φ.
2. Deduce an algorithm to compute $[\![\mathsf{E}\mathsf{G}\,\varphi]\!]$ from M and $[\![\varphi]\!]$. What is the complexity of your procedure?

Exercise 4 (CTL$^+$)**.** CTL$^+$ extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

$$f ::= \top \mid a \mid f \wedge g \mid \neg f \mid \mathsf{E}\,\varphi \mid \mathsf{A}\,\varphi \qquad \text{(state formulæ } f, g\text{)}$$
$$\varphi ::= \varphi \wedge \psi \mid \neg\varphi \mid \mathsf{X}\,f \mid f\,\mathsf{U}\,g \qquad \text{(path formulæ } \varphi, \psi\text{)}$$

where a is an atomic proposition. The associated semantics is that of CTL*.

We want to prove that, for any CTL$^+$ formula, there exists an equivalent CTL formula.

1. Give an equivalent CTL formula for

$$\mathsf{E}((a_1 \mathbin{\mathsf{U}} b_1) \wedge (a_2 \mathbin{\mathsf{U}} b_2)) .$$

2. Generalize your translation for any formula of form

$$\mathsf{E}\left(\bigwedge_{i=1,\dots,n}(\psi_i \mathbin{\mathsf{U}} \psi'_i) \wedge \mathsf{G}\,\varphi\right) . \tag{1}$$

What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL$^+$ formula:

$$\mathsf{E}(\mathsf{X}\,a \wedge (b \mathbin{\mathsf{U}} c)) .$$

4. Using subformulæ of form (1) and E modalities, give an equivalent CTL formula to

$$\mathsf{E}(\mathsf{X}\,\varphi \wedge \bigwedge_{i=1,\dots,n}(\psi_i \mathbin{\mathsf{U}} \psi'_i) \wedge \mathsf{G}\,\varphi') . \tag{2}$$

What is the complexity of your translation?

5. We only have to transform any CTL$^+$ formula into (nested) disjuncts of form (2). Detail this translation for the following formula:

$$\mathsf{A}((\mathsf{F}\,a \vee \mathsf{X}\,a \vee \mathsf{X}\,\neg b \vee \mathsf{F}\,\neg d) \wedge (d \mathbin{\mathsf{U}} \neg c)) .$$