TD 1: Models

Exercise 1 (Rendez-vous with Data). Consider the synchronization of transition systems with variables through a rendez-vous mechanism. Such a system is of form $M = (S, \Sigma, V, (D_v)_{v \in V}, T, I, AP, l)$ where $V$ the set of (typed) variables $v$, each with domain $D_v$.

We want to extend the rendez-vous mechanism between systems with variables with the ability to exchange data values. For instance, a system $M_i$ may transmit a value $m$ by performing

$$s_i \xrightarrow{! m} s'_i,$$

only if some system $M_j$ is ready to receive the message, i.e. to perform

$$s_j \xrightarrow{? v} s'_j,$$

where $v$ is a variable of $M_j$ and $m$ is in $D_v$. Of course the synchronization is also possible if $M_j$ performs instead

$$s_j \xrightarrow{? m} s'_j.$$

1. Propose Structural Operational Semantics for the rendez-vous with data synchronization.

2. Assume $D_v = D$ for all variables $v$ in $V$.

   Generalize these semantics to allow sending and receiving terms in $T(\Sigma, V)$ built from the variables and a finite set of symbols $\Sigma$ that contains $D$.

Exercise 2 (Needham-Schroeder Protocol). We consider the analysis of a public-key authentication protocol proposed by Needham and Schroeder in 1978. The protocol relies on

- the generation of *nounces* $N_C$: random numbers that should only be used in a single session, and
- on public key encryption: we denote the encryption of message $M$ using $C$’s public key by $(M)_C$.

A(lice) and B(ob) try to make sure of each other’s identity by the following (very simplified) exchange:
1. Alice first presents herself (the $A$ part of the message) and challenges Bob with her nounce $N_A$. Assuming both cryptography and random number generation to be perfect, only Bob can decrypt $\langle A, N_A \rangle_B$ and find the correct number $N_A$.

2. Bob responds by proving his identity (the $N_A$ part) and challenges Alice with his own nounce $N_B$.

3. Finally, Alice proves her identity by sending $N_B$.

The nounces $N_A$ and $N_B$ are used by Alice and Bob as secret keys for their communications.

In order to account for the insecure channel, we have to add an intruder $I$ to the model, who has his own nounce $N_I$, and can read and send any message it fancies, but can only decrypt $\langle M \rangle_I$ messages and cannot guess the nounces generated by Alice and Bob.

We can model the behaviour of Alice as a transition system $M_A$ with variables and rendez-vous with data, using a single variable $N$ ranging over $D_N = \{N_A, N_B, N_I\}$.

1. Provide a model $M_B$ for Bob.

2. Provide a model $M_I$ the intruder.

3. Unfold an execution path in the synchronized product of $M_A$, $M_B$, and $M_I$ that unveils a flaw in the protocol.
Exercise 3 (Channel Systems). The course notes present the semantics of FIFO channels. We consider here the case of a single finite system $M = \langle S, \Sigma, T, I, AP, \ell \rangle$ along with $n$ unbounded channels over a finite set $\Gamma$ (i.e. each channel is declared as $c_i : \text{channel}[\infty]$ of $\Gamma$ for each $1 \leq i \leq n$). Configurations of the full system $M$ are thus in $S \times (\Gamma^*)^n$, i.e. of form $(s, \gamma_1, \ldots, \gamma_n)$ where $s$ is a state of $S$ and channel $i$ contains $\gamma_i$.

Without loss of generality, we consider the channels to be empty in the initial configurations, i.e. $I = \{(s_i, \varepsilon, \ldots, \varepsilon) \mid s_i \in I\}$.

We are interested in the control-state reachability problem, i.e. given an $n$-channel system $M$ and a state $s$, does there exist an initial state $s_i$ in $I$ and $n$ strings $\gamma_1, \ldots, \gamma_n$ in $\Gamma^*$ s.t. $(s_i, \varepsilon, \ldots, \varepsilon) \rightarrow^* (s, \gamma_1, \ldots, \gamma_n)$?

1. Consider the case $\Gamma = \{a\}$ and $n = 1$. Show that the control-state reachability problem is decidable in PTIME.
2. Show that it becomes undecidable for $n = 1$ and $|\Gamma| \geq 2$.
3. We allow the channel systems to test the contents of a channel for emptiness:

$$\nu(c_j) = \varepsilon \land s_i \xrightarrow{\text{empty}(c_j)} s'_i \quad (\bar{s}, \nu) \xrightarrow{\text{empty}(c_j)} (\bar{s}', \nu)$$

Show that the control-state reachability problem is then undecidable for $n \geq 2$ even if $|\Gamma| = 1$. 

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