Logical and Computational Structures for Linguistic Modeling
Part 3 – Mildly Context-Sensitive Formalisms

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Part I

Tree Adjoining Grammars

Extending CFG with structures

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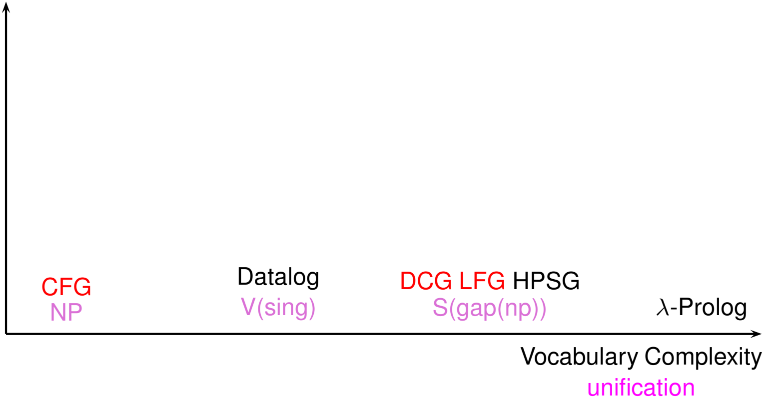
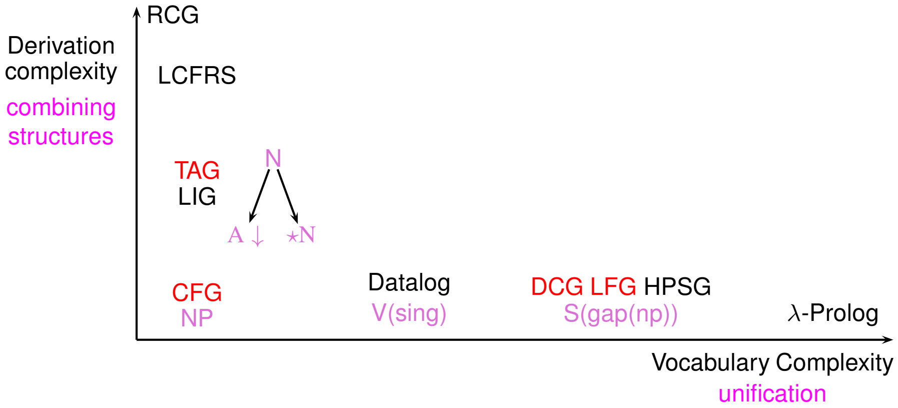


Extending CFG with structures



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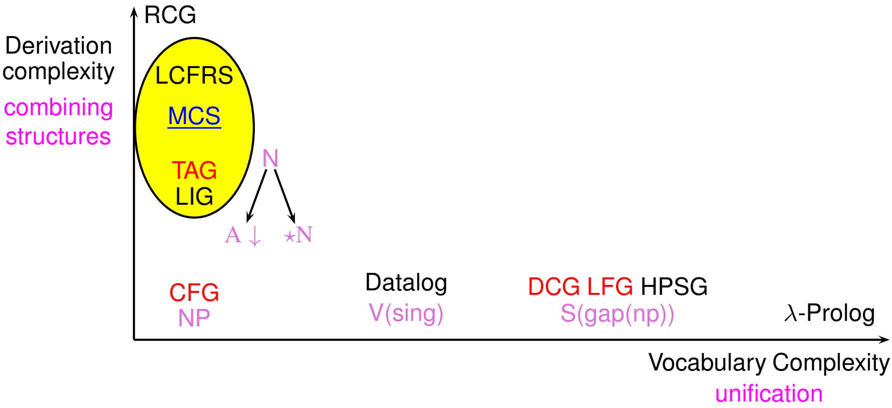
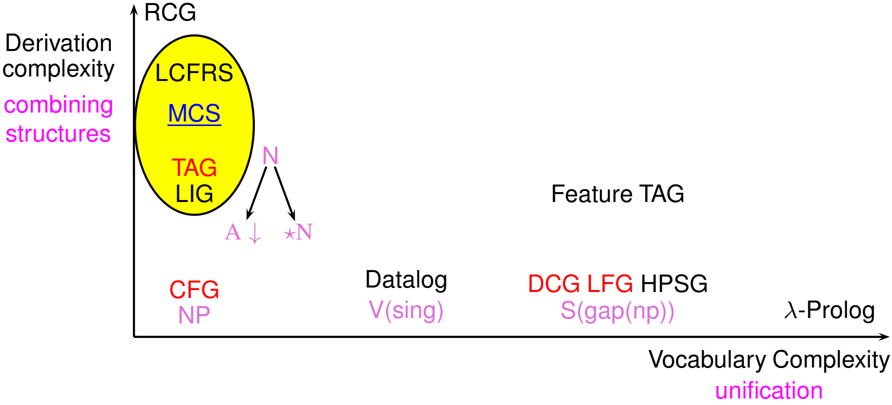


Extending CFG with structures



Outline

1. Some background about TAGs
2. Deductive chart-based TAG parsing
3. Automata-based tabular TAG parsing

From CFG to Tree Substitution Grammars

```
s   --> np vp
np  --> pn
np  --> det n
np  --> np pp
vp  --> v np
vp  --> vp pp
pp  --> prep
  np
```

CFG productions:

- are too local
  $\Longrightarrow$ need decorations for info propagation
- are generally not lexicalized
  but info often propagated from words
  also more efficient parsing algo for lexicalized grammars

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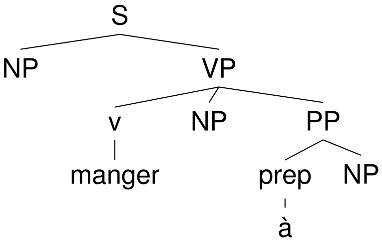
CFG productions can be grouped into trees
$\Longrightarrow$ we get Tree Substitution Grammars (TSG)

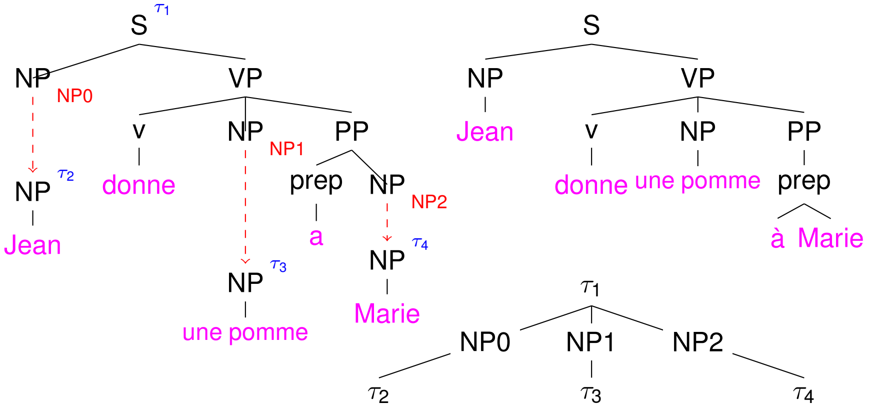
For instance, dealing with ditransitive verb **donner**



Derivation Tree vs Parse Tree

TSG are strongly equivalent to CFG

However, for TSG, parse trees and derivation trees are not equivalent



Furthermore, several derivations may lead to a same parse tree

One step farther: adjoining

How to deal with:

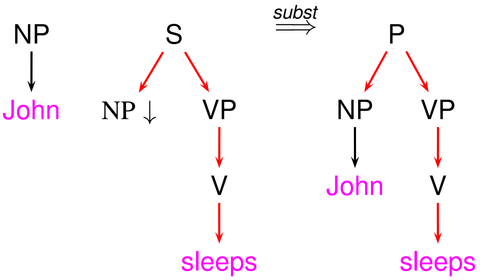
Jean donne souvent une pomme à Marie

Need a way to insert the adverb somewhere in the verbal tree
$\Longrightarrow$ adjoining operation

$\leadsto$ Tree Adjoining Grammars

TAG: a small example

Tree Adjoining Grammars [TAGs] [Joshi] build parse trees from initial and auxiliary trees by using 2 tree operations: substitution and adjoining



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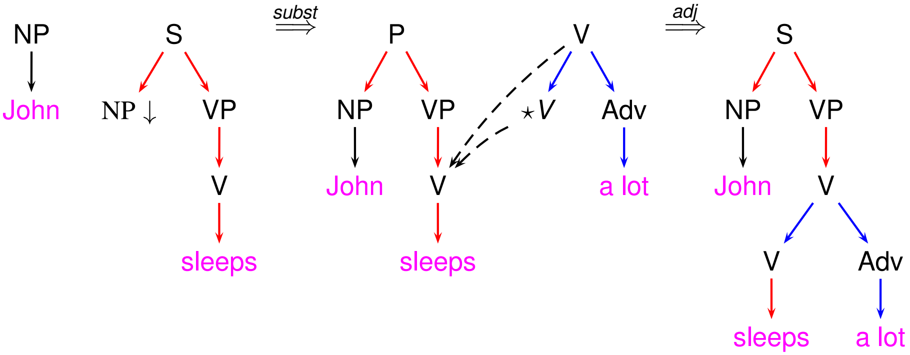
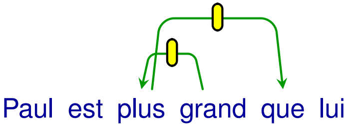


A more complex example: French comparative



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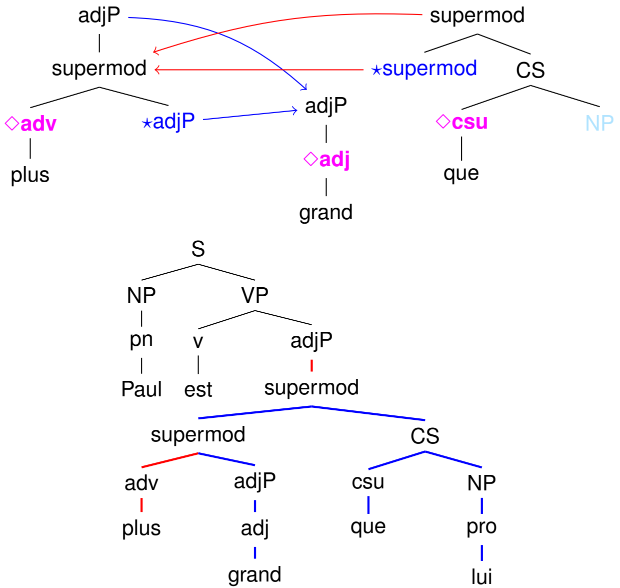


TAGs: (First) Formal definition

A TAG G is a tuple $(\mathcal{N}, \Sigma, S, \mathcal{I}, \mathcal{A})$ where

- Σ a finite set of terminal symbols
- $\mathcal{N}$ a finite set of non-terminal symbols
- $S \in \mathcal{N}$ the axiom
- $\mathcal{I}$ and $\mathcal{A}$ are two finite sets of elementary trees over $\mathcal{N} \cup \Sigma \cup \{\epsilon\}$
  only leaf nodes ν may have a label $l(\nu) \in \Sigma \cup \{\epsilon\}$
  - the trees α in $\mathcal{I}$ are initial trees
  - the trees β in $\mathcal{A}$ are auxiliary trees and have a unique leaf node marked $(\star)$ as a foot f_β with same label than the root node r_β, i.e. $l(f_\beta) = l(r_\beta)$

Two operations may be used to combine the elementary trees

- substitution of a leaf node ν of γ by some initial tree $\alpha \in \mathcal{I}$, $(l(\nu) = l(t_\alpha))$
- adjoining of an (internal) node ν of γ by some auxiliary tree $\beta \in \mathcal{A}$

G generates a tree language and a string language

$$T(G) = \{\gamma | \alpha \Longrightarrow^\star \gamma \wedge \mathrm{yield}(\gamma) \in \Sigma^\star \wedge \alpha \in \mathcal{I} \wedge r_\alpha = S\}$$

$$L(G) = \{\mathrm{yield}(\gamma) | \gamma \in T(G)\}$$

Adjoining

Assuming $\gamma = (V, E)$ with $\nu \in V$ and $\beta = (V_\beta, E_\beta)$ with $r_\beta, f_\beta \in V_\beta$, such that $l(\nu) = l(r_\beta) \in \mathcal{N}$

$$\gamma[adj(\nu, \beta)] = (V', E')$$

with

$$\begin{cases} V' = V \cup V_\beta \backslash \{\nu\} \\ E' = \bigcup \begin{cases} \{(x,y) \in E | x \neq \nu \wedge y \neq \nu\} \\ E_\beta \\ \{(x, r_\beta) | (x,\nu) \in E\} \\ \{(f_\beta, y) | (\nu, y) \in E\} \end{cases} \end{cases}$$

Note: The node sets are assumed to be renamed to avoid clashes, i.e. $E \cap E' = \emptyset$

Adjoining contraints

A full definition of TAGs should include constraints on adjoining nodes:

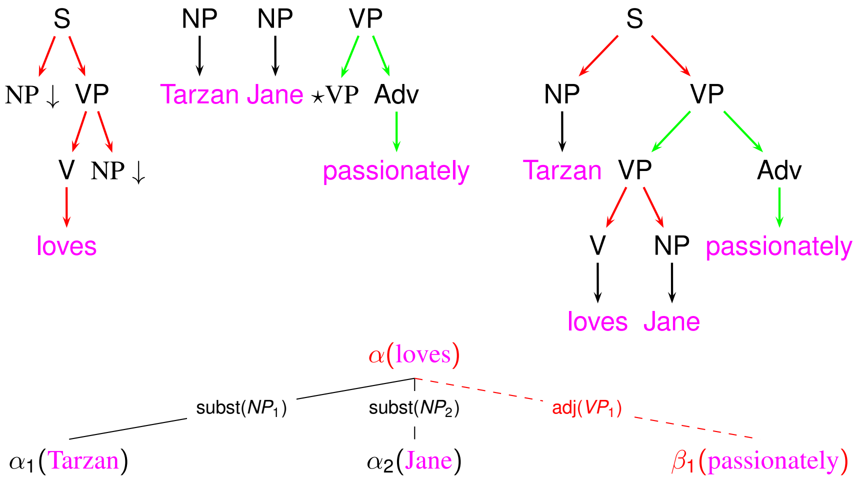
A TAG G is a tuple $(\mathcal{N}, \Sigma, S, \mathcal{I}, \mathcal{A}, f_{OA}, f_{SA})$ where,
assuming V set of nodes in $\mathcal{I} \cup \mathcal{A}$,

- $f_{OA} : V \mapsto \{0, 1\}$ specify if adjoining on ν is obligatory (1) or not (0)
- $f_{SA} : V \mapsto 2^{\mathcal{A}}$ specify which auxiliary trees may be adjoined on ν
  note: ν becomes non-adjoinable with $f_{SA}(\nu) = \emptyset$

Adjoining constraints necessary for getting the full expressive power of TAGs but they are often implicit:

- no adjoining on leaf nodes (including foot nodes)
- explicit mandatory adjoining (MA, $+$) marks on some nodes
- explicit non adjoining (NA, $-$) marks on some nodes

Derivation tree



For TAGs, derivation tree not isomorphic to parse tree
but close from semantic level.

Regular Tree Languages

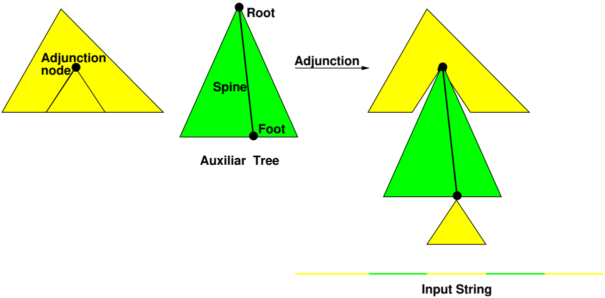
For a TAG G, its set of derivation trees $D(G)$ forms a regular tree language

i.e., $D(G)$ may be generated by a finite tree automaton
(top-down) term rewrite rules of the form

$$q_0 \leftarrow a(q_1, \ldots, q_n), \; q_i \in \mathcal{Q}, a \in \mathcal{F}$$

may also be seen as the parse trees for some CFG G'

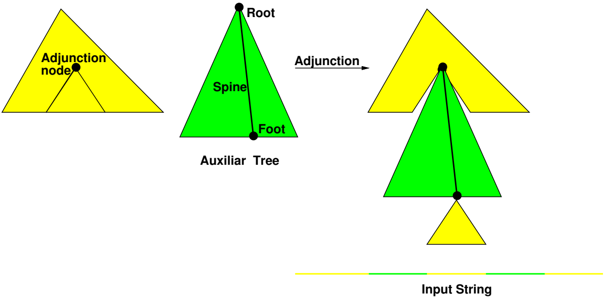
TAG complexity: Adjoining



adjoining

- discontinuity (hole in aux tree)
- crossing (both sides of the hole)

TAG complexity: Adjoining



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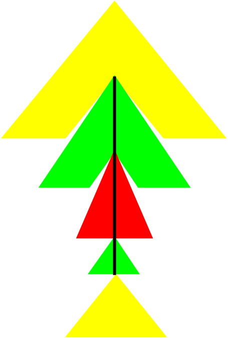


nested adjoining

- discontinuity (hole in aux tree)
- crossing (both sides of the hole)
- unbounded synchronization (both sides of spine)

Expressive power of TAGs

The adjoining operation extends the expressive power of TAGs w.r.t. CFGs.

- long distance dependencies (wh-pronoun extraction for instance)
- crossed dependencies as given by copy language "ww" or by language "$a^n b^n c^n$"

(1) omdat ik Cecillia de nijlpaarden zag voeren
because I Cecilia the hippopotamuses saw feed
because I saw Cecilia feed the hippopotamuses

Expressive power (limits)

TAGs can't handle the following languages:

- $a^n b^m c^n d^m e^n f^m$
- multiple copy languages w^n with $n > 2$.

Pumping lemma

Tree Adjoining Languages satisfy a pumping lemma

If L is a TAL, then there exists N, such for all $w \in L$ and $|w| > N$, there exist $x, y, z, v_1, v_2, w_1, w_2, w_3, w_4 \in \Sigma^\star$, such that

$$\begin{cases} |v_1 v_2 w_1 w_2 w_3 w_4| \leq N \\ |w_1 w_2 w_3 w_4| \geq 1 \end{cases}$$

and one of the following case holds

1. $w = xw_1 v_1 w_2 y w_3 v_2 w_4 z$ and $\forall k \geq 0$, $xw_1^k v_1 w_2^k y w_3^k v_2 w_4^k z \in L$
2. $w = xw_1 v_1 w_2 v_2 w_3 y w_4 z$ and $\forall k \geq 0$, $xw_1^{k+1} v_1 w_2 v_2 w_3 (w_2 w_4 w_3)^k y w_4 z \in L$
3. $w = xw_1 y w_2 v_1 w_3 v_2 w_4 z$ and $forallk \geq 0$, $xw_1 y (w_2 w_1 w_3)^k w_2 v_1 w_3 v_2 w_4^{k+1} \in L$

Closure properties of TALs

As CFLs, TALs form an Abstract Family of Languages (AFL):

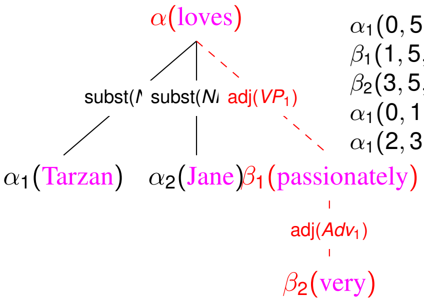
1. closed by intersection with regular languages
2. closed by union, concatenation, and Kleene-iteration
3. closed by homomorphism and inverse homomorphism

In particular, (1) $\Longrightarrow$ notion of Shared Derivation Forest

Shared Derivation Forests

Formal definition in Vijay-Shanker & Weir 1993

$_0$ *Tarzan* $_1$ *loves* $_2$ *Jane* $_3$ *very* $_4$ *passionately* $_5$



$\alpha_1(0,5) \rightarrow \alpha_1(0,1)$ loves$(1,2)$ $\alpha_2(2,3)$ $\beta_1(1,5,1,3)$
$\beta_1(1,5,1,3) \rightarrow \beta_2(3,5,4,5)$ passionately$(4,5)$
$\beta_2(3,5,4,5) \leftarrow$ very$(3,4)$
$\alpha_1(0,1) \leftarrow$ Tarzan$(0,1)$
$\alpha_1(2,3) \leftarrow$ Janes$(0,1)$

More formally, use tree nodes rather than trees
Space complexity in $O(n^6)$ by binarization (adj on spine node ν)

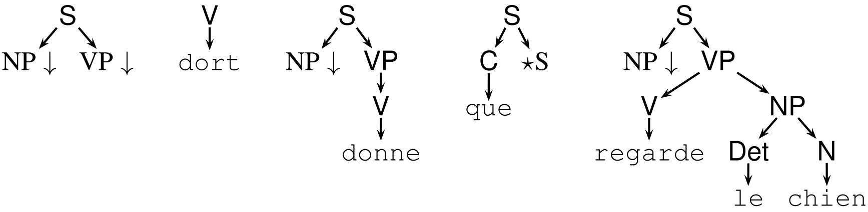
$$\nu^\top(i,j,r,s) \rightarrow r_\beta^\top(i,j,p,q)\ \nu^\perp(p,q,r,s)$$

Well formed trees

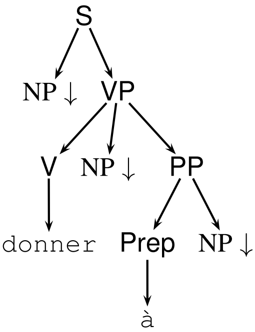
Many possible ways to define elementary trees
In practive, elementary trees follow some linguistic principles:

- lexical anchoring: at least, one non-empty lexical (frontier) node the head (or anchor)
- sub-categorization: a frontier node for each argument sub-categorized by the head
  domain of locality
- semantic consistency: a tree correspond to the scope of a semantic predicate with its arguments
- non-composition: a tree stands for a single semantic unit

A few bad trees:

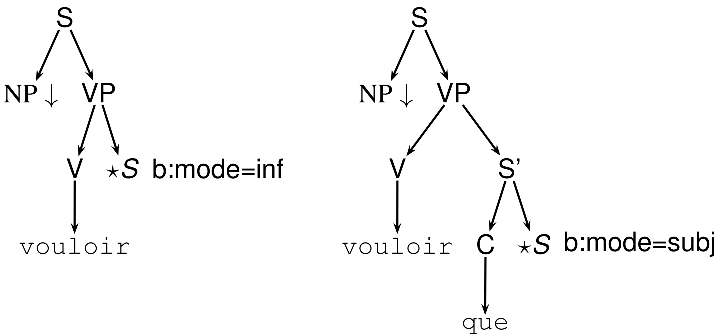


Subcategorization



Feature TAGs

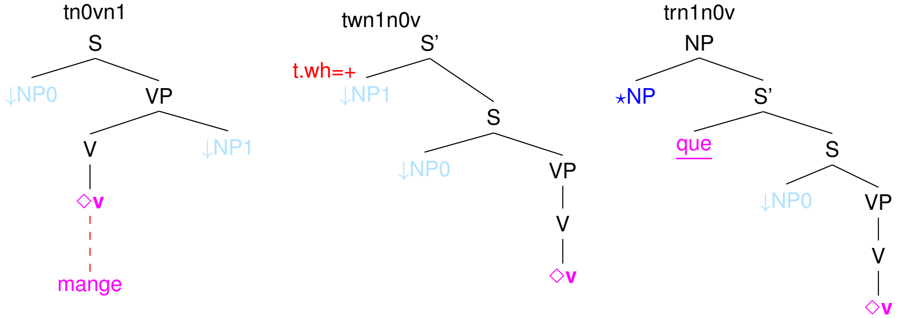
The nodes may be decorated with a pair (top, bot) of decorations



When adj on ν, unification of $\nu.top$ with $r_\beta.top$ and $\nu.bot$ with $f_\beta.bot$
alternate way to express adjoining constraints

Note: for flat decorations, same expressive power and complexity

Trees derived from a canonical ones grouped into families
e.g. family of transitive verbs



+ all other extractions (on NP_0) + passive + extractions on passive
+ ordering + multiple realizations + . . .

⇝ XTAG architecture

- a set of trees (with anchor nodes) grouped into families
- a lexicon $\mathcal{L}$ specifying for each word *w* the set of families it may anchor + additional constraints

Meta-grammars

Large coverage TAG $\Longrightarrow$ many trees to write and maintain !

Alternative: generate the trees from a higher description level: meta-grammars Abeillé, Candito

- hierarchy of classes, containing constraints
  A precedes B, A dominates B, ...
- a class deals with a linguistic facet
  e.g. verb argument, refined into subject or object
- a class may require or provide functionalities
- the classes may be combined to form neutral classes
- the constraints of the neutral classes used to generate elementary trees

$\Longrightarrow$ used for FRMG, a large-coverage French TAG
`http://alpage.inria.fr/frmgwiki`
(plus mechanisms for factorizing elementary trees)

Long-distance dependencies

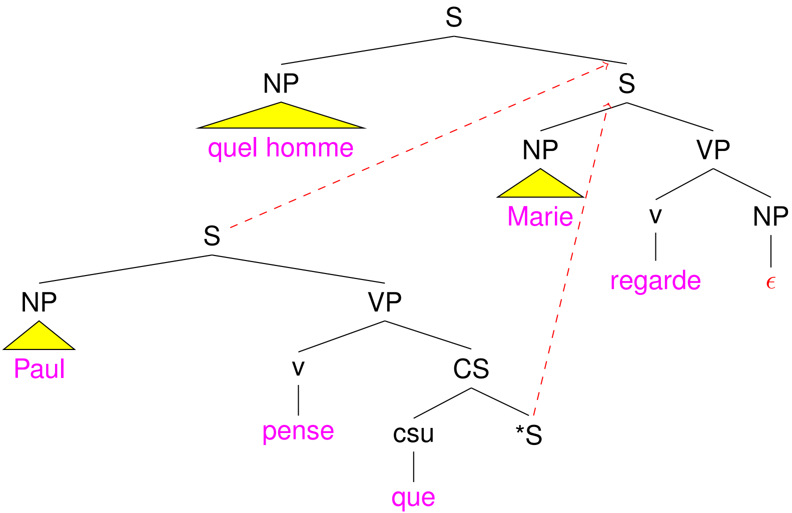
(Recursive) Adjoining may replace LFG's functional uncertainty for long-distance dependencies

Jean demande [quel homme Paul pense [que Marie regarde ϵ]]

$$
\begin{array}{lll}
\text{S'} \longrightarrow & \text{NP} & \text{S} \\
 & (\downarrow\text{Wh}) =_c + & \uparrow=\downarrow \\
 & (\uparrow\text{Focus}) = \uparrow & (\downarrow\text{Wh})=+ \\
 & (\uparrow\text{Focus})=\uparrow(\mathrm{Comp})^{\star}\mathrm{Obj} &
\end{array}
$$

Long-distance dependencies (TAGs)

Handled through repeated adjoining



Outline

Deductive parsing

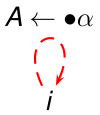
Formalization of chart parsing

Use of

- universe of tabulable **items**, representing (set of) partial parses
- items often build upon **dotted rules** $A_0 \leftarrow A_1 \dots A_i \bullet A_{i+1} \dots A_n$
- chart edges labeled by dotted rules (items $\equiv \langle i, j, A \leftarrow \alpha \bullet \beta \rangle$)
- a deductive system specifying how to derive items

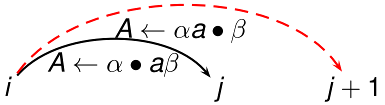
CKY as a deductive system (for CFGs)

$$\frac{}{\langle i, i, A \leftarrow \bullet\alpha \rangle} \qquad \exists A \leftarrow \alpha$$



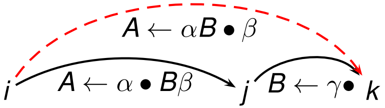
(Seed)

$$\frac{\langle i, j, A \leftarrow \alpha \bullet a\beta \rangle}{\langle i, j+1, A \leftarrow \alpha a \bullet \beta \rangle} \qquad a = a_{j+1}$$



(Scan)

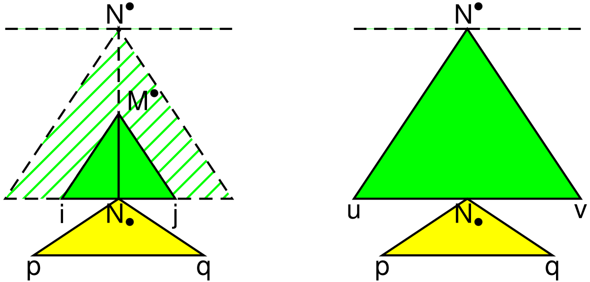
$$\frac{\langle i, j, A \leftarrow \alpha \bullet B\beta \rangle \quad \langle j, k, B \leftarrow \gamma\bullet \rangle}{\langle i, k, A \leftarrow \alpha B \bullet \beta \rangle}$$



(Complete)

CKY algorithm for TAGs [Vijay-Shanker & Joshi 85]
Presentation:

- Dotted trees $N^\bullet$ and $N_\bullet$ where N is a node of an elementary tree
- Items $\langle N^\bullet, i, p, q, j\rangle$ and $\langle N_\bullet, i, p, q, j\rangle$ with p, q possibly covering a foot node.



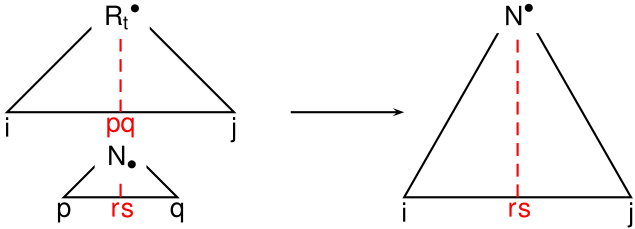
$\langle M_\bullet, i, p, q, j\rangle$

Without adjoining: $\langle N_\bullet, p, -, -, q\rangle$
With adjoining: $\langle N^\bullet, u, -, -, v\rangle$

Rule (Adjoin)

Gluing a sub-tree at a foot node.

$$\frac{\langle \mathrm{N}_{\bullet}, p, r, s, q\rangle \;\; \langle \mathrm{R_t}^{\bullet}, i, p, q, j\rangle}{\langle \mathrm{N}^{\bullet}, i, r, s, j\rangle} \qquad \mathrm{label}(N) = \mathrm{label}(R_t) \qquad \text{(Adjoin)}$$



Rule (NoAdjoin)

When no adjoining on a node

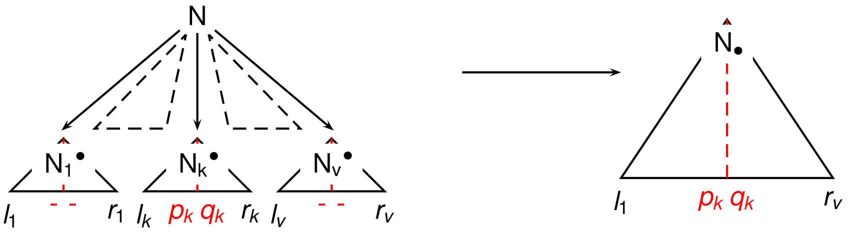
$$\frac{\langle \mathrm{N}_{\bullet}, p, r, s, q \rangle}{\langle \mathrm{N}^{\bullet}, p, r, s, q \rangle} \qquad \text{(NoAdjoin)}$$

Rule (Complete)

Gluing all node's children

$$\frac{\langle N_i^{\bullet}, l_i, p_i, q_i, r_i \rangle_{i=1,\ldots,v}}{\langle N_{\bullet}, l_1, \cup p_i, \cup q_i, r_v \rangle} \quad \begin{array}{c} N \\ \swarrow \quad \downarrow \quad \searrow \\ N_1 \quad \ldots \quad N_v \end{array} \quad \text{and } \forall i,\ l_{i+1} = r_i \qquad \text{(Complete)}$$

Note: At most one child (k) covers a foot node with $(\cup p_i, \cup q_i) = (p_k, q_k)$



Complexity

Other deductive rules needed to handle

1. substitution
2. terminal scanning

+ axioms

Time complexity $O(n^{\max(6,1+v+2)})$ with

- v : maximal number of children per node
- 2 : number of indexes to cover a possible unique foot node

Normalization using binary-branching trees ($v = 2$) $\Longrightarrow$ complexity $O(n^6)$

4 indexes per item $\Longrightarrow$ Space complexity in $O(n^4)$ for a recognizer
$O(n^6)$ for a parser, keeping backpointers to parents

Optimal worst-case complexities
but practically, even less efficient than CKY for CFGs

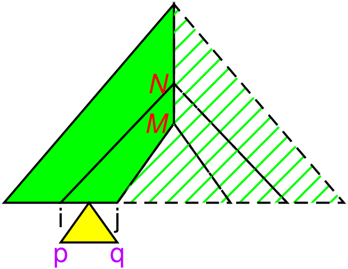
Prediction, dotted trees and dotted producctions

To mark prediction, new dotted trees [Shabes]: $^{\bullet}$N and $_{\bullet}$N

Alternative: equivalence with dotted productions

$$\begin{tikzcd} & N \arrow[dl] \arrow[d] \arrow[dr] & \\ N_1 & \ldots & N_v \end{tikzcd} \equiv N \leftarrow N_1 \ldots N_v$$

dotted tree	dotted production
$N_k^{\bullet}$, $^{\bullet}N_{k+1}$	$N \leftarrow N_1 \ldots N_k \bullet N_{k+1} \ldots N_v$
$^{\bullet}$R (root)	$\top \leftarrow \bullet R$
R$^{\bullet}$ (root)	$\top \leftarrow R \bullet$
$_{\bullet}$N	$N \leftarrow \bullet N_1 \ldots N_v$
N$_{\bullet}$	$N \leftarrow N_1 \ldots N_n \bullet$



$$\langle N \leftarrow \alpha \bullet M\beta, i, p, q, j \rangle$$

Non prefix valid Earley algorithm

- Glue a sub-tree at foot node F_t (maybe useless !)

$$\frac{\langle M \leftarrow \gamma\bullet, p, r, s, q\rangle \;\; \langle \top \leftarrow R_t\bullet, i, p, q, j\rangle}{\langle M \leftarrow \gamma\bullet, i, r, s, j\rangle} \quad \mathrm{label}(M) = \mathrm{label}(R_t) \qquad \text{(Adjoin)}$$

- Advance in recognition of N's children

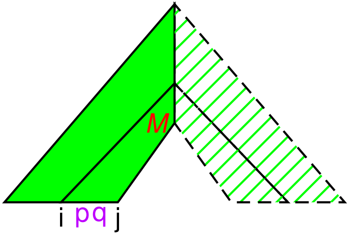
$$\frac{\langle N \leftarrow \alpha \bullet M\beta, i, u, v, j\rangle \;\; \langle M \leftarrow \gamma\bullet, j, r, s, k\rangle}{\langle N \leftarrow \alpha M \bullet \beta, i, u \cup r, v \cup s, k\rangle} \qquad \text{(Complete)}$$

(Adjoin) and (Complete) similar to CKY (binary form)

Adjoining Prediction

Predict adjoining at M

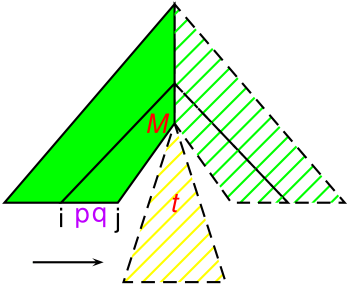
$$\frac{\langle N \leftarrow \alpha \bullet M\beta, i, p, q, j\rangle}{} \qquad \text{label}(M) = \text{label}(R_t) \qquad \text{(CallAdj)}$$



Adjoining Prediction

Predict adjoining at M

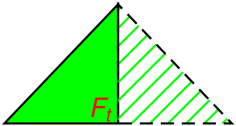
$$\frac{\langle N \leftarrow \alpha \bullet M\beta, i, p, q, j\rangle}{\langle \top \leftarrow \bullet R_t, j, -, -, j\rangle} \qquad \text{label}(M) = \text{label}(R_t) \qquad \text{(CallAdj)}$$



Foot Prediction

Predict a sub-tree root at M to recognize below foot node F_t

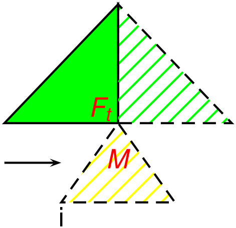
$$\frac{\langle F_t \leftarrow \bullet \perp, i, -, -, i\rangle}{} \qquad \mathrm{label}(F_t) = \mathrm{label}(M) \qquad \text{(CallFoot)}$$



Foot Prediction

Predict a sub-tree root at M to recognize below foot node F_t

$$\frac{\langle F_t \leftarrow \bullet \perp, i, -, -, i\rangle}{\langle M \leftarrow \bullet\gamma, i, -, -, i\rangle} \qquad \text{label}(F_t) = \text{label}(M) \qquad \text{(CallFoot)}$$



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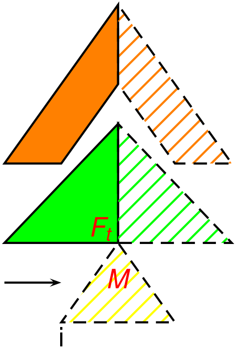


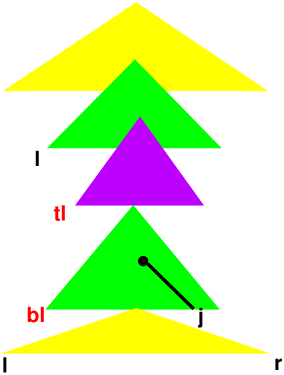
The prediction of M not related to the node M' having triggered the adjoining of t
$\Longrightarrow$ Non prefix valid parsing strategy

Complexity

- Space complexity remains $O(n^4)$
- Dotted productions $\Longrightarrow$ implicit binarization $\Longrightarrow$ time in $O(n^6)$
- Non prefix valid: impact difficult to evaluate in practice
- **Note**: Dotted productions also applicable to improve CKY

Prefix valid Early [Shabes]

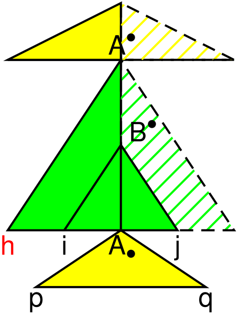
Complexities time in $O(n^9)$ and space in $O(n^6)$ due to 6-index items



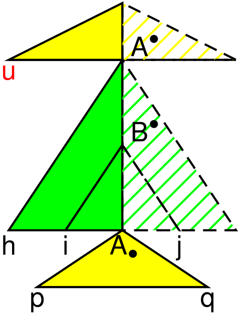
Actually, *tl* and *bl* may be avoided using dotted productions

Prefix valid Earley [Nederhof]

Item with only an extra index h: $\langle h, N \leftarrow \alpha \bullet \beta, i, p, q, j\rangle$
h states starting (leftmost) position of current tree



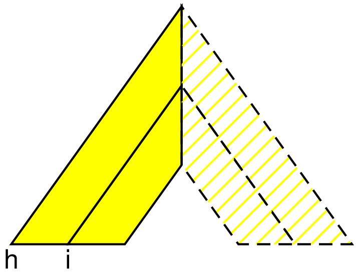
$\langle h, N \leftarrow \alpha B \bullet \beta, i, p, q, j\rangle$



$\langle u, A \leftarrow \gamma \bullet, p, -, -, q\rangle$

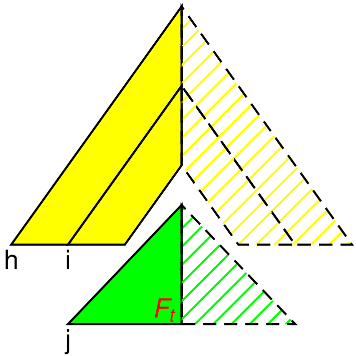
Foot prediction

$$\frac{\langle h, N \leftarrow \alpha \bullet M\beta, i, p, q, j \rangle}{} \quad \text{label}(F_t) = \text{label}(M) \qquad \text{(CallFootPf)}$$



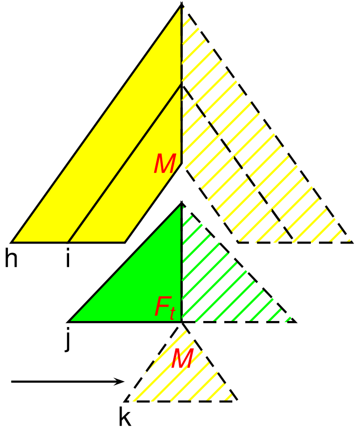
Foot prediction

$$\frac{\begin{array}{c}\langle h, N \leftarrow \alpha \bullet M\beta, i, {\color{purple}p}, {\color{purple}q}, j\rangle \\ \langle j, F_t \leftarrow \bullet \perp, k, {\color{purple}-}, {\color{purple}-}, k\rangle\end{array}}{} \quad \mathrm{label}(F_t) = \mathrm{label}(M) \qquad \text{(CallFootPf)}$$



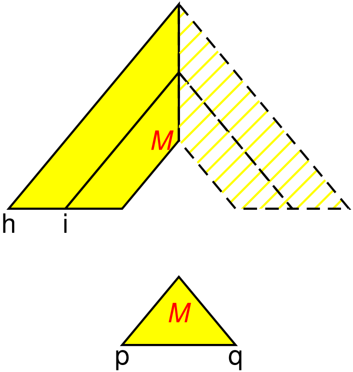
Foot prediction

$$\frac{\begin{array}{c}\langle h, N \leftarrow \alpha \bullet M\beta, i, p, q, j\rangle \\ \langle j, F_t \leftarrow \bullet \perp, k, -, -, k\rangle\end{array}}{\langle h, M \leftarrow \bullet\gamma, k, -, -, k\rangle} \qquad \text{label}(F_t) = \text{label}(M) \qquad \text{(CallFootPf)}$$



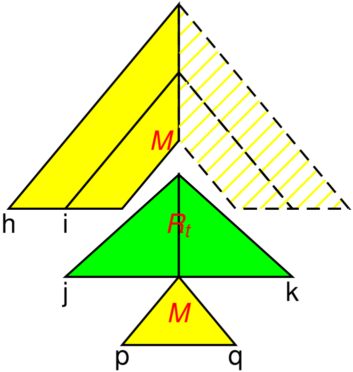
Adjoining return

$$\frac{\begin{array}{c}\langle h, N \leftarrow \alpha \bullet M\beta, i, u, v, j\rangle \\ \langle h, M \leftarrow \gamma\bullet, p, r, s, q\rangle\end{array}}{} \qquad \text{label}(M) = \text{label}(R_t) \qquad \text{(AdjoinPf)}$$



Adjoining return

$$\frac{\begin{array}{c}\langle h, N \leftarrow \alpha \bullet M\beta, i, u, v, j\rangle \\ \langle j, \top \leftarrow R_t \bullet, j, p, q, k\rangle \\ \langle h, M \leftarrow \gamma \bullet, p, r, s, q\rangle\end{array}}{} \quad \mathrm{label}(M) = \mathrm{label}(R_t) \qquad \text{(AdjoinPf)}$$



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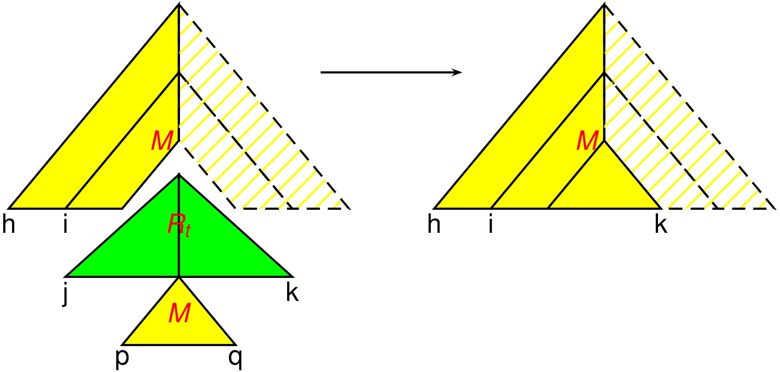


Raw complexity

Maximal time complexity provided by (AdjoinPf) : $O(n^{10})$ because of 10 indexes

$$\frac{\begin{array}{c}\langle h, N \leftarrow \alpha \bullet M\beta, i, u, v, j\rangle \\ \langle j, \top \leftarrow R_t\bullet, j, p, q, k\rangle \\ \langle h, M \leftarrow \gamma\bullet, p, r, s, q\rangle\end{array}}{\langle h, N \leftarrow \alpha M \bullet \beta, i, u \cup r, v \cup s, k\rangle} \quad \mathrm{label}(M) = \mathrm{label}(R_t) \qquad \text{(AdjoinPf)}$$

But (u, v) or (r, s) equals $(-, -)$
$\Longrightarrow$ (Case analysis) splitting rule into 2 sub-rules $\Longrightarrow O(n^8) \Longrightarrow$ not sufficient !

Splitting and intermediary structures

Split (AdjoinPf) into 2 successive steps with an intermediary structure

$$[M \leftarrow \gamma \bullet, j, r, s, k]$$

This intermediary structure combines the aux. tree with the subtree rooted at M

$$\frac{\begin{array}{c}\langle j, \top \leftarrow R_t \bullet, j, p, q, k\rangle \\ \langle h, M \leftarrow \gamma \bullet, p, r, s, q\rangle\end{array}}{[M \leftarrow \gamma \bullet, j, r, s, k]} \qquad \text{(AdjoinPf-1)}$$

$$\frac{\begin{array}{c}\langle h, N \leftarrow \alpha \bullet M\beta, i, u, v, j\rangle \\ [M \leftarrow \gamma \bullet, j, r, s, k] \\ \langle h, M \leftarrow \gamma \bullet, p, r, s, q\rangle\end{array}}{\langle h, N \leftarrow \alpha M \bullet \beta, i, u \cup r, v \cup s, k\rangle} \qquad \text{(AdjoinPf-2)}$$

Projection

$$\frac{\begin{array}{c}\langle j, \top \leftarrow R_t \bullet, j, p, q, k\rangle \\ \langle h, M \leftarrow \gamma \bullet, p, r, s, q\rangle\end{array}}{[M \leftarrow \gamma \bullet, j, r, s, k]} \quad \text{(AdjoinPf-1)}$$

Involves 7 indexes $\{j, p, q, k, h, r, s\}$ but h not consulted

$$\frac{\langle h, M \leftarrow \gamma \bullet, p, r, s, q\rangle}{\langle \star, M \leftarrow \gamma \bullet, p, r, s, q\rangle} \quad \text{(Proj)}$$

$$\frac{\begin{array}{c}\langle j, \top \leftarrow R_t \bullet, j, p, q, k\rangle \\ \langle \star, M \leftarrow \gamma \bullet, p, r, s, q\rangle\end{array}}{[M \leftarrow \gamma \bullet, j, r, s, k]} \quad \text{(AdjoinPf-1)}$$

Finally, $O(n^6)$ time complexity

Case of (AdjoinPf-2)

$$\frac{\begin{array}{c}\langle h, N \leftarrow \alpha \bullet M\beta, i, u, v, j\rangle \\ [M \leftarrow \gamma\bullet, j, r, s, k] \\ \langle h, M \leftarrow \gamma\bullet, p, r, s, q\rangle\end{array}}{\langle h, N \leftarrow \alpha M \bullet \beta, i, u \cup r, v \cup s, k\rangle} \quad \text{(AdjoinPf-2)}$$

10 indexes $\Longrightarrow$ Raw complexity in $O(n^{10})$

At least one pair in (u, v) or (r, s) equals $(-, -)$;
Case splitting $\Longrightarrow O(n^8)$

Pair (p,q) not consulted; projection $\Longrightarrow O(n^6)$

Preliminary conclusion

Rule splitting, intermediary structures, and projections decrease complexities but increase the number of steps
To be practically validated !

Designing a tabular algorithm for TAGs is complex!

- Designing items
- Understanding the invariants
- Formulating the deductive rules (simultaneously handling tabulation and strategy)
- Optimizing rules (splitting and projections)

How to adapt for close formalisms such as Linear Indexed Grammars [LIG]?

$$A_0([\circ \circ x]) \leftarrow A_1([]) \dots A_k([\circ \circ y]) \dots A_n([])$$

LIG

Indexed Grammars: Context-Free grammars with non terminals decorated with stacks
Linear Indexed Grammars: a single stack propagated per production

A LIG $G = (\mathcal{N}, \Sigma, \mathcal{I}, S, \mathcal{P})$ where

- $\mathcal{I}$ is a finite set of indices
- $\mathcal{P}$ is a finite set of productions of the form

  $$A[\circ\circ\alpha] \rightarrow A_1[] \ldots A_i[\circ\circ\beta] \ldots A_n[]$$

  or

  $$A[] \rightarrow \gamma$$

  with $\gamma \in \Sigma^\star$ and $\alpha, \beta \in \mathcal{I}^\star$

Relationship with (linear monadic) Context-Free Tree Languages

LIGs and TAGs

LIGs and TAGs are weakly equivalent, and almost strongly equivalent

TAGs may be easily encoded by LIGs, using tree nodes as non-terminals

- adjoining node ν in γ using aux. tree β

$$\nu[\circ\circ] \rightarrow r_\beta[\circ\circ\,\nu]$$

- discharging a node ν with children $\nu_1, \ldots, \nu_n$ at a foot node f_β

$$f_\beta[\circ\circ\,\nu] \leftarrow \nu_1[\alpha_1] \ldots \nu_n[\alpha_n]$$

  where $\alpha_i = [\circ\circ]$ if ν_i on spine, and $\alpha_i = []$ otherwise
- traversing a node ν without adjoining

$$\nu[\circ\circ] \leftarrow \nu_1[\alpha_1] \ldots \nu_n[\alpha_n]$$

  with same conditions on α_i than above

Reverse way more difficult: no locality constraint between push and pop points (same aux. tree β for TAGs)

Suggest using LPDAs to parse LIGs and TAGs
but non efficient and non termination

Outline

From formalisms to automata

Methodology:

- Automata are operational devices
  used to describe the steps of Parsing Strategies
- Dynamic Programming interpretations of automata used to identify context-free subderivations that may be tabulated.

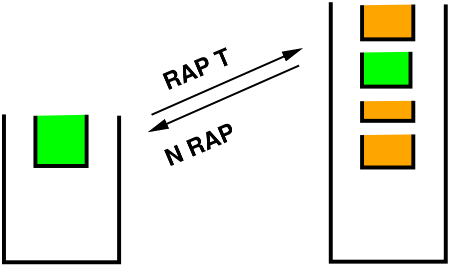
Formalisms	Automata	Tabulation	Notes
RegExp	FSA	-	
CFG	PDA	$O(n^3)$	Lang
TAG / LIG	2-Stack Automata	$O(n^6)$	Becker, Clergerie & Pardo
	Embedded PDA	$O(n^6)$	Nederhof

Problem: 2-stack automata (or EPDA) have the power of Turing Machine (intuition) moving left- or rightward $\equiv$ pushing on first or second stack & popping the other one
$\Longrightarrow$ need restrictions

EPDA

Embedded Push-Down Automata Becker are natural candidates for LIGs (and TAGs) by handling stack of stacks.

Two flavors: Top-Down and Bottom-Up EPDAs



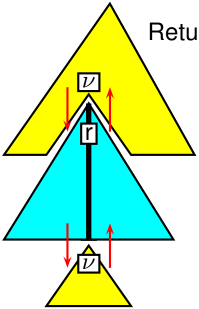
Solution: stack asymmetry

Master Stack: to keep trace of uncompleted tree traversals

Auxiliary Stack: only to keep trace of uncompleted adjunctions
Adjunction info: (top-down) $\overline{\nu}^n = \nu$ and (bottom-up) $\underline{\nu}_n = \perp$
$^\bullet$T, T$^\bullet$, $^\bullet$B, B$^\bullet$: prediction and propagation info about top and bottom node decorations (Feature TAGs)

Calls (top-down prediction)

Returns (bottom-up propagation)



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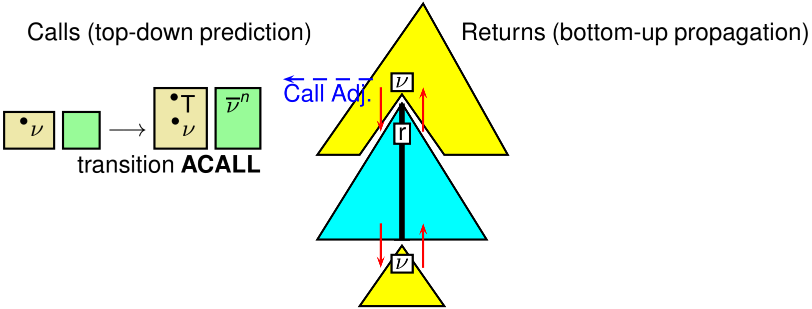

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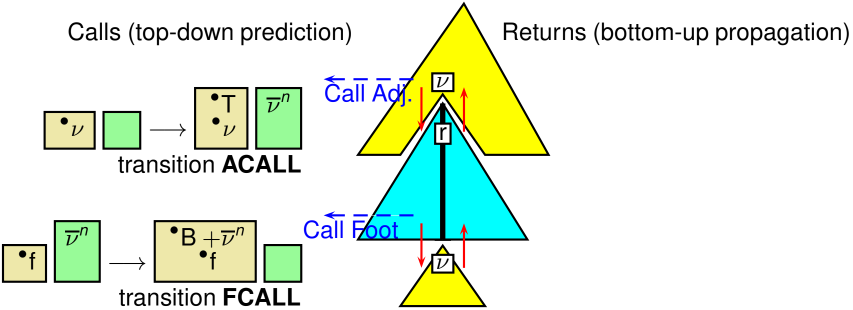

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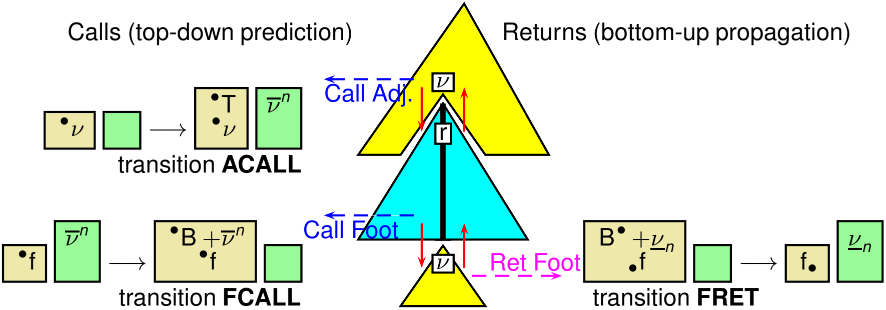

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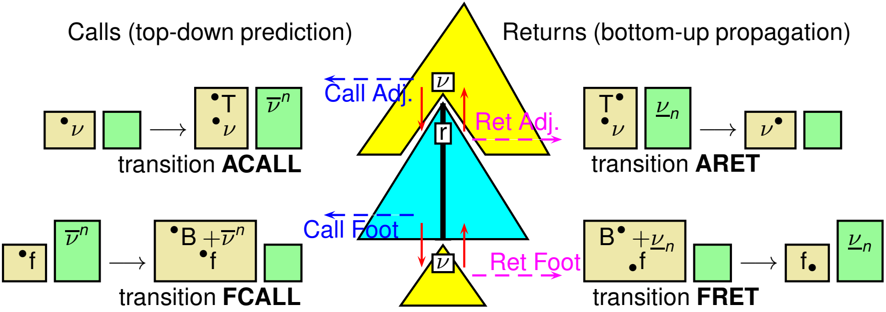
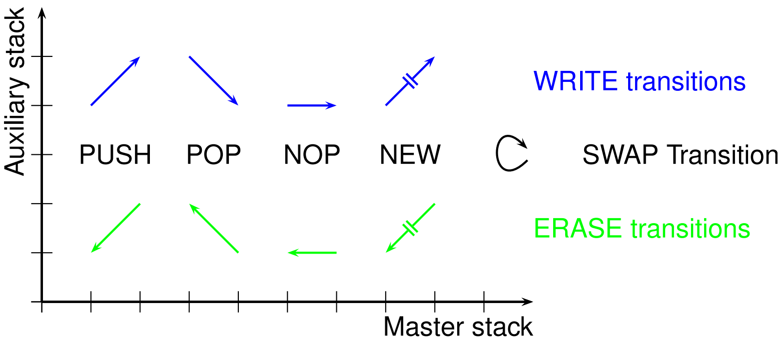


Transitions

Retracing in erase mode concerns only the size of **AS** (not its content).
Retracing possible because :

> *WRITE transitions leave marks (PUSH, POP, NOP, NEW) in the Master Stack that can only be removed by a dual ERASE transition.*



Context-Free derivation for PDAs

Dynamic Programming : Recursive decomposition of problems into elementary subproblems that may be combined, tabulated, and reused
eg the knapsack problem

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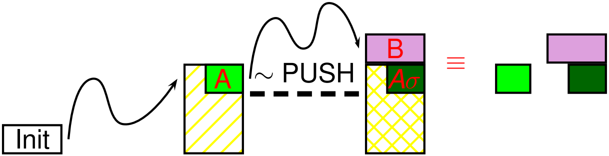
For PDAs, derivations broken into elementary Context-Free sub-derivations:



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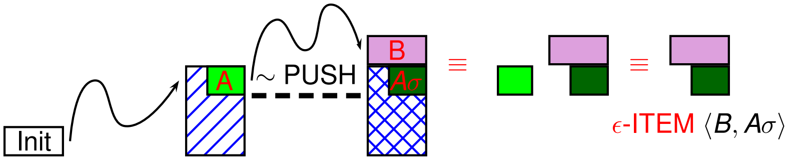

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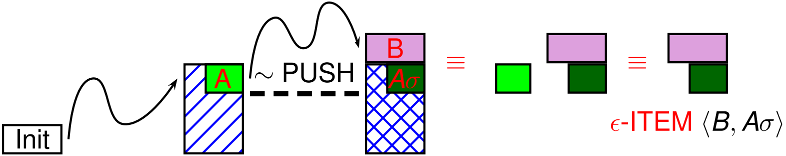

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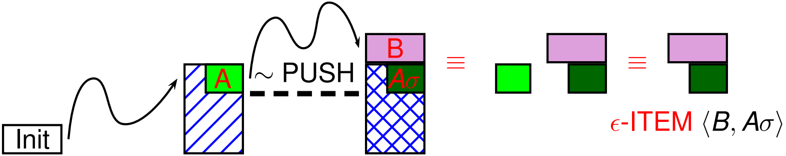
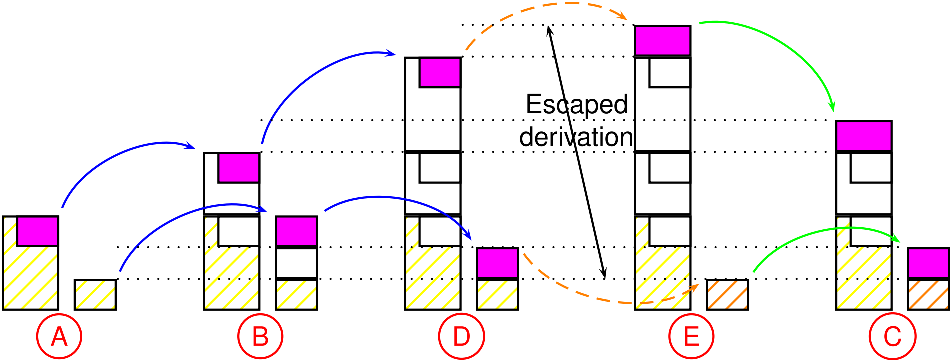


A is the fraction ϵ of information consulted to trigger the subderivation and not propagated to B.

(Escaped) CF derivations for 2SA



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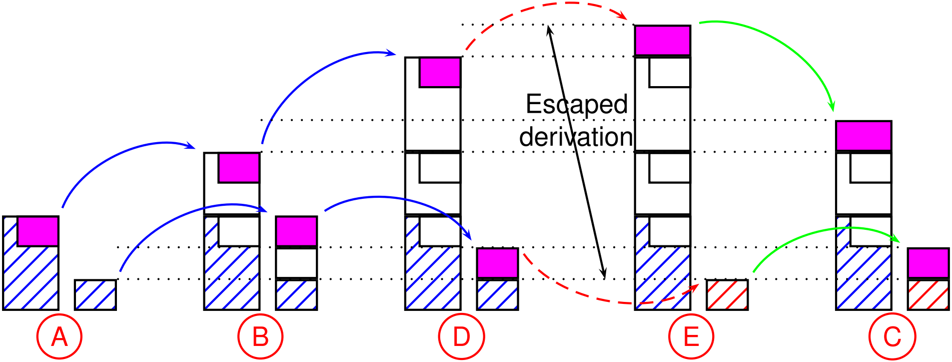
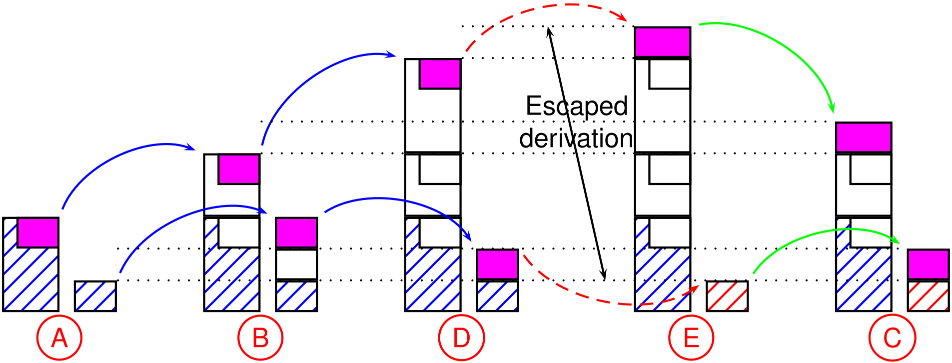


$\Longrightarrow$ 5-point xCF items $AB[DE]C = \langle\epsilon A\rangle\langle\epsilon B, b\rangle[\langle\epsilon D, d\rangle\langle E\rangle]\langle C, c\rangle$
[TAG] $\rightsquigarrow \langle\epsilon A\rangle\langle\epsilon B\rangle[\langle\epsilon D\rangle\langle E\rangle]\langle C\rangle$

When no escaped part $\Longrightarrow$ 3-point CF items $ABC = \langle\epsilon A\rangle\langle\epsilon B, b\rangle\langle C\rangle$

(new generalization) escaped part $[DE]$ may take place between A and B

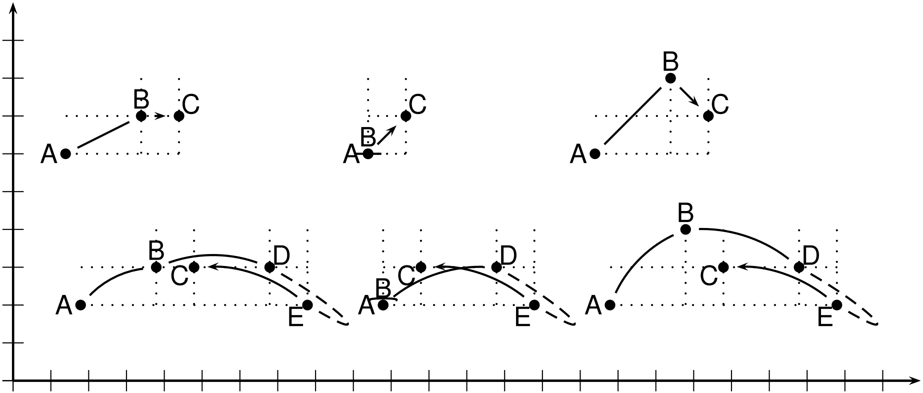
xCFs and TAGs



- *A* root of elementary tree
- *B* start of adjoining
- *C* current position in the tree
- *D* and *E* left and right borders of the foot

Item shapes

At most 5 indexes per items $\Longrightarrow$ Space complexity in $O(n^5)$
SD-2SA restrictions & transition kinds $\Longrightarrow$ 6 possible item shapes



Combining items and transitions

By graphically playing with items and transitions, we find 10 composition rules with $O(n^8)$ time complexity
may be split into 11 rules with $O(n^6)$ time complexity

(Easy:) Write a POP mark: $I_1 + I_2 + \tau = I_3$

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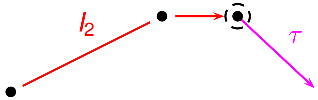

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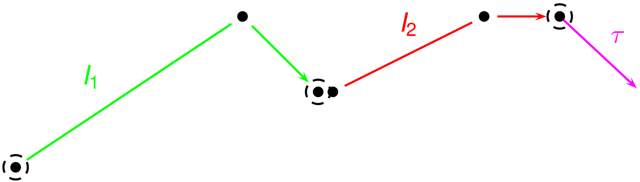

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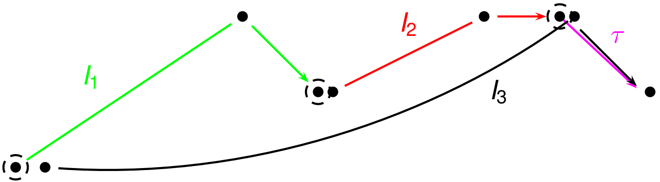

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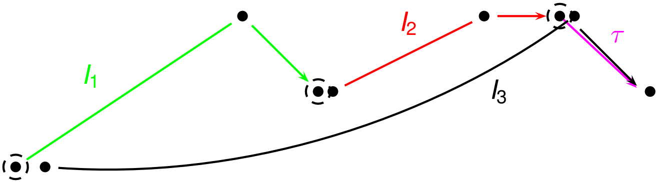

Consultation of 3 indexes ⊙ $\Longrightarrow$ Complexity $O(n^3)$

Combining items and transitions (2)

(complex:) Erasing a PUSH mark: $I_1 + I_2 + I_3 + \tau = I_4$
e.g. when returning from auxiliary tree (ending adjoining)

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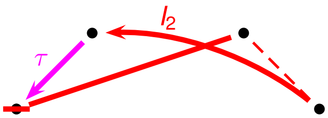

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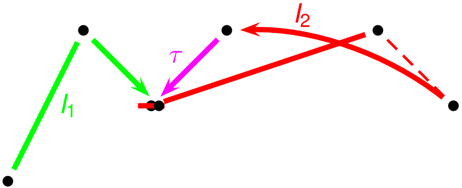

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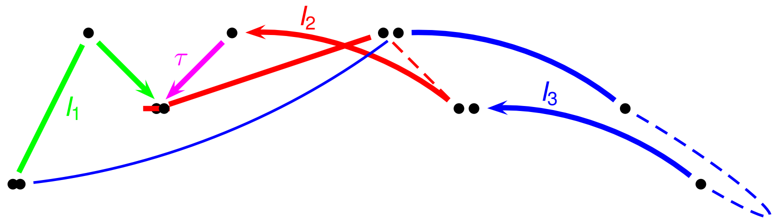

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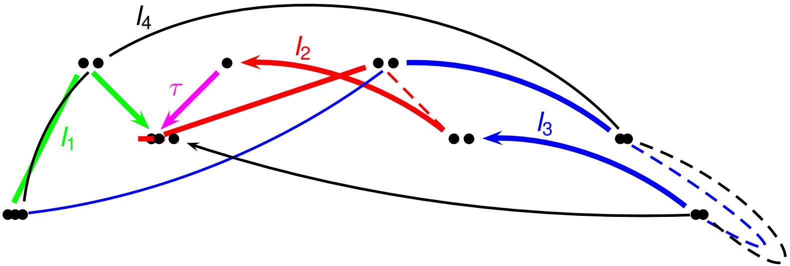

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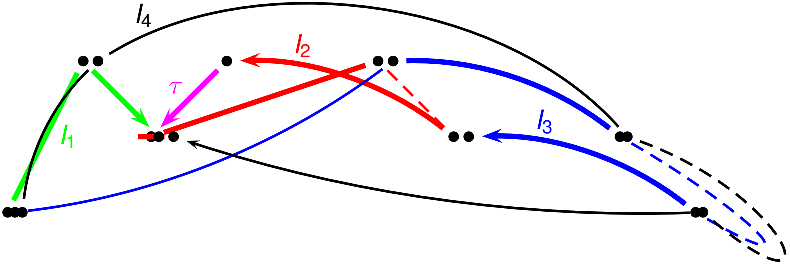
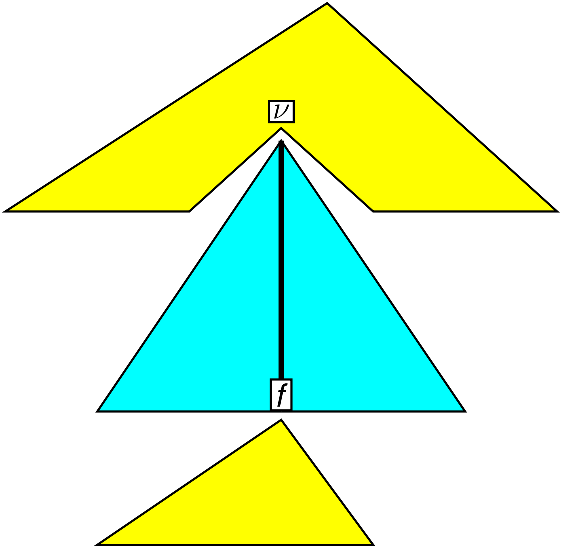


- Consultation of 8 indexes $\Longrightarrow$ Complexity $O(n^8)$
- need to decompose, project and use intermediary steps (as seen before)

Cascade of partial evaluations



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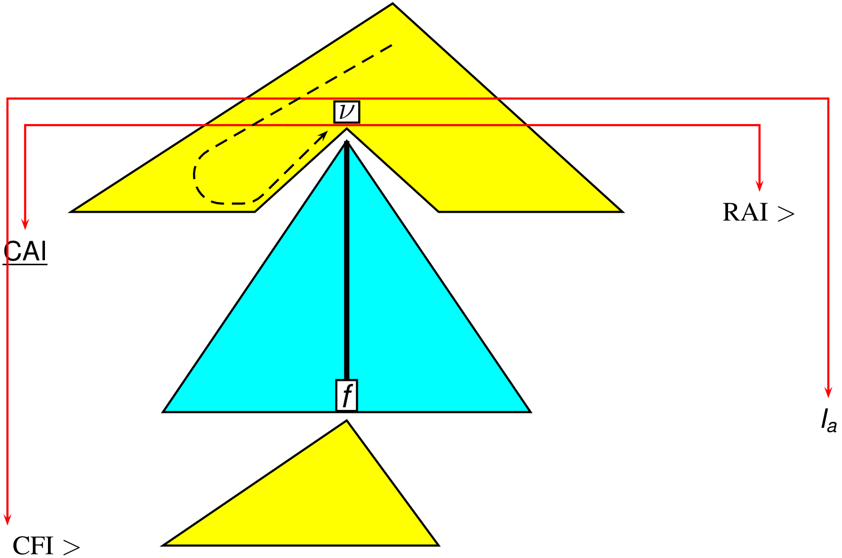

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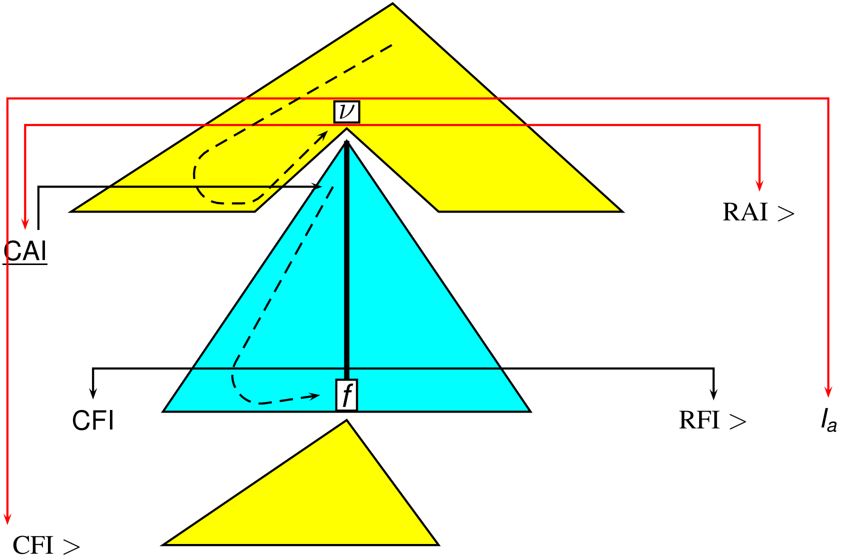

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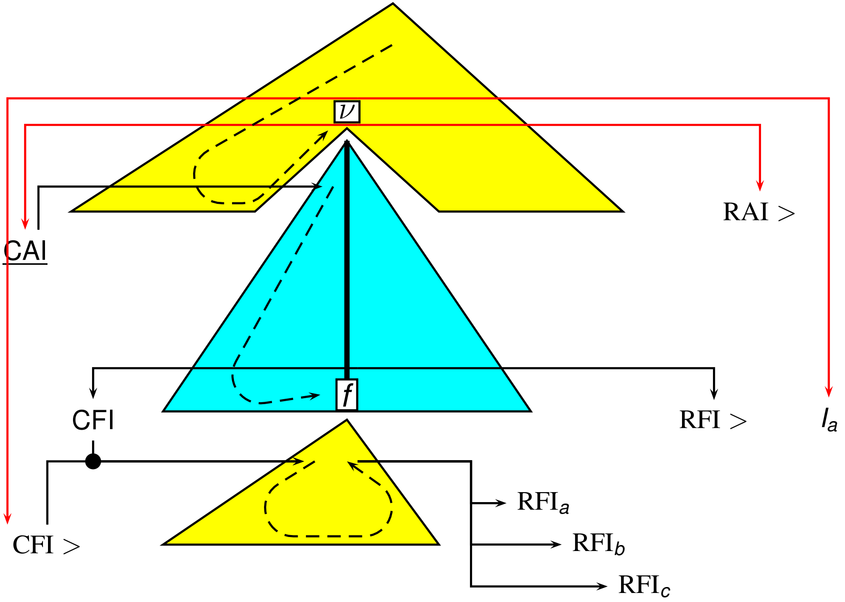

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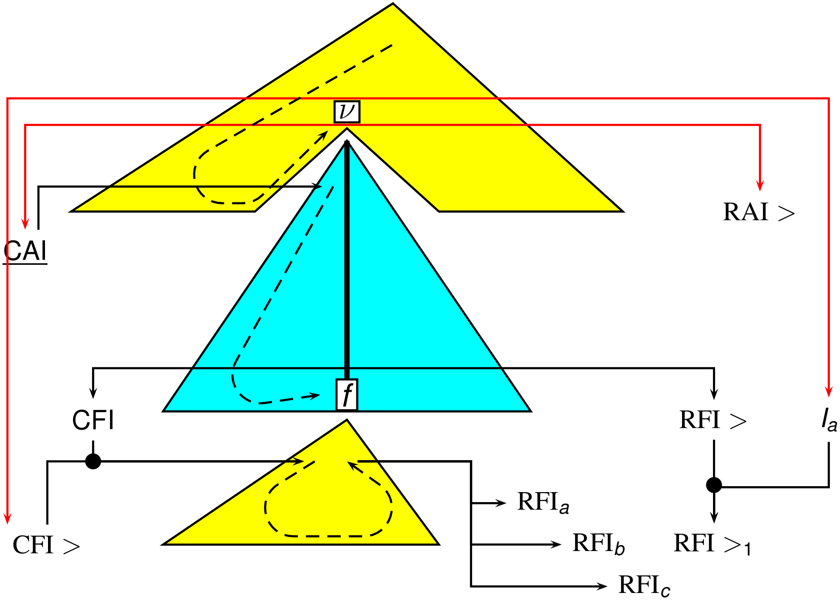

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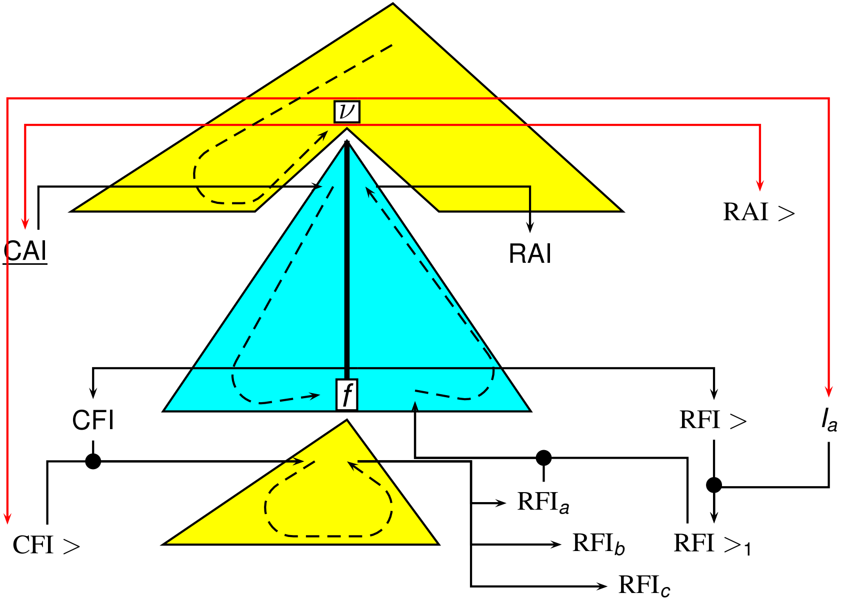

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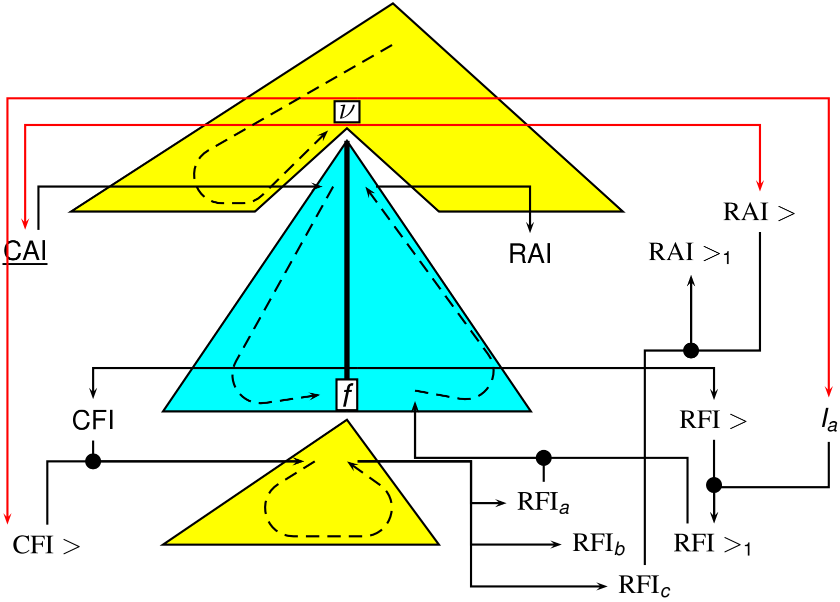

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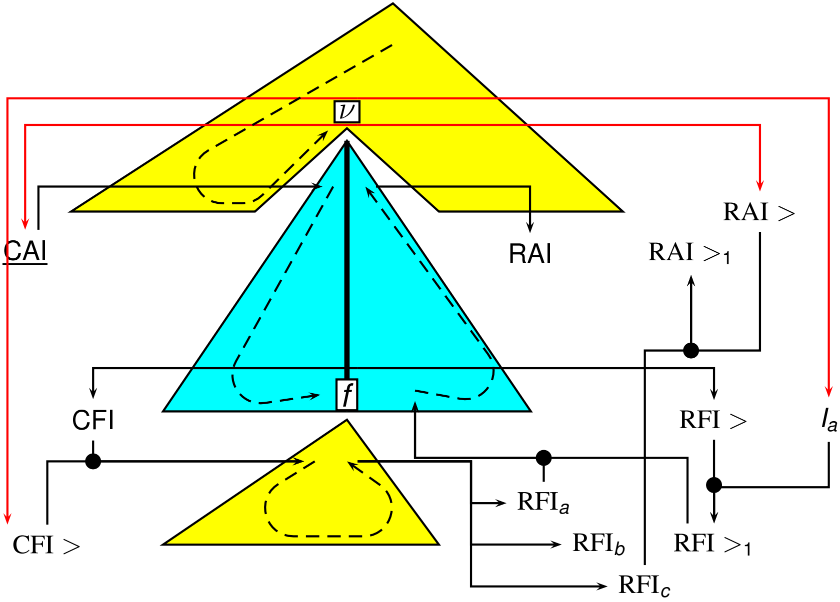

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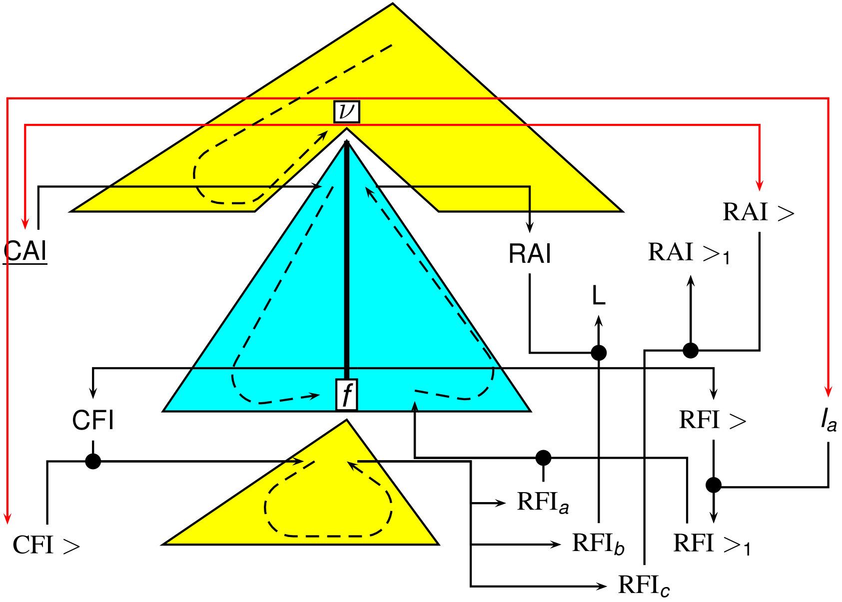

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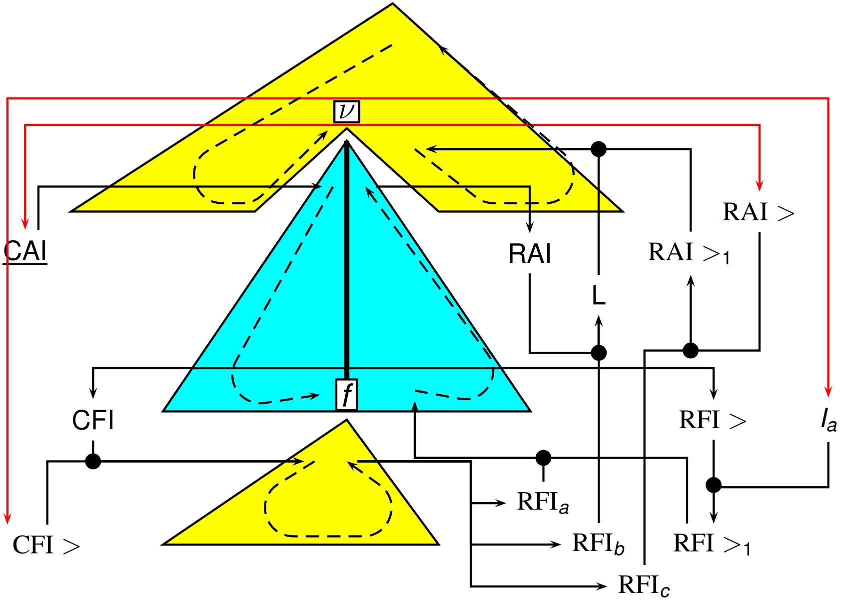

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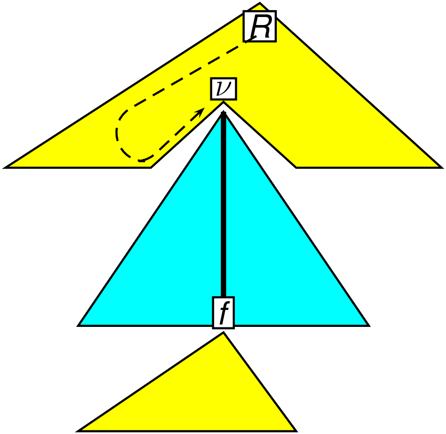

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- But more efficient in practice !
- And suggesting extensions, based on the idea of continuation

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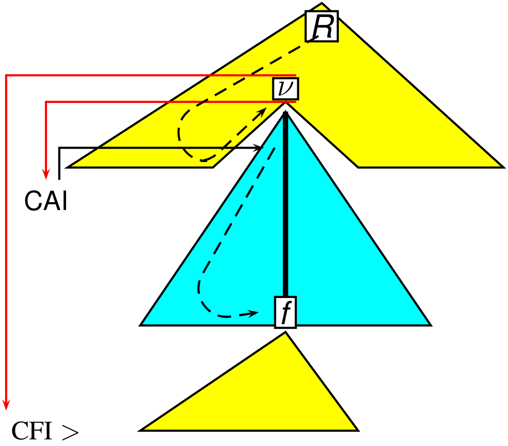

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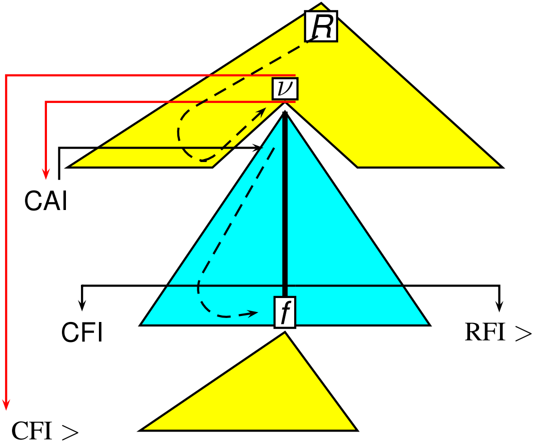

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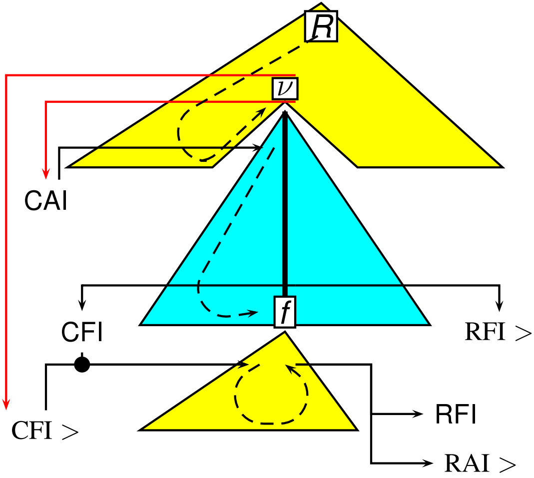

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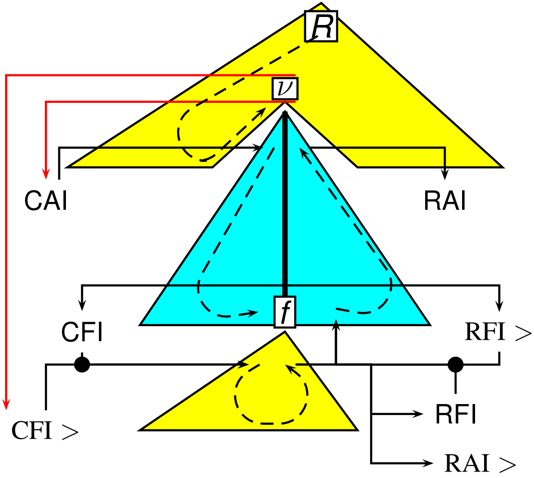

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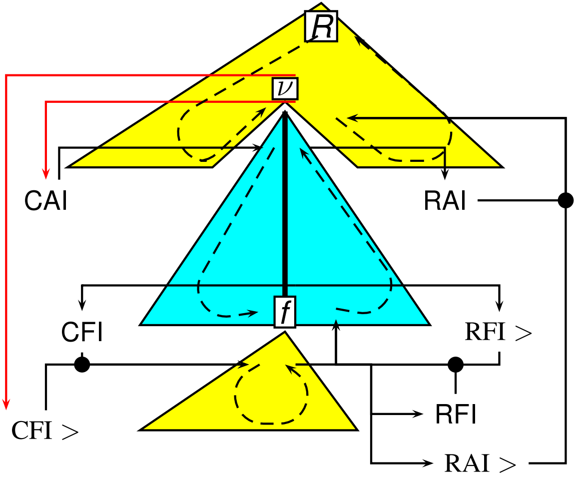

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Part II

MCS in general

Outline

4 Thread Automata and MCS formalisms

5 A Dynamic Programming interpretation for TAs

Mildly Context Sensitivity

An informal notion covering formalisms such that:

- they are powerful enough to model crossing, such as $a^n b^n c^n$
- they are parsable with polynomial complexity
  i.e. Given L, there exists k, membership $w \in$ checked in $O(|w|^k)$
- they generate string languages satisfying the constant growth property

  $$\exists G, G \text{ finite }, \exists n_0, \forall w \in \mathcal{L}, |w| > n_0 \Longrightarrow \exists g \in G, \exists w' \in \mathcal{L}, |w| = |w'| + g$$

  (intuition) the languages are generated by finite sets of generators

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Some MCS languages:

- TAGs and LIGs
- Local Multi Component TAGs (MC-TAGs Weir)
- Linear Context-Free Rewriting Systems (LCFRS Weir)
- Simple Range Concatenation Grammars (sRCG Boullier)

Semi-linearity

The Constant Growth property subsumed by stronger semi-linearity under Parikh image

The Parikh image of $w \in \{a_1, \ldots, a_n\}^\star$ defined as $p(w) = (|w|_{a_1}, \ldots, |w|_{a_n})$

The Parikh image of L defined as $p(L) = \{p(w) | w \in L\}$

A set V of vectors over $\mathbb{N}^\Sigma$ is linear is generated by a base $v_0, v_1, \ldots, v_n \in \mathbb{N}^\Sigma$ by

$$V = \{v_0 + \Sigma_{i=1}^n k_i v_i | k_i \in \mathbb{N}\}$$

V is semilinear if $V = \bigcup_{i=1}^k V_i$ is a finite union of linear sets V_i

A language L is semilinear if $p(L)$ is semilinear

(intuition) A MCS language is generated, modulo some permutations, by a finite set of generators

MCS: discontinuity and interleaving

Discontinuous interleaved constituents present in linguistic phenomena
Nesting, Crossing, Topicalization, Deep extraction, Complex Word-Order ...

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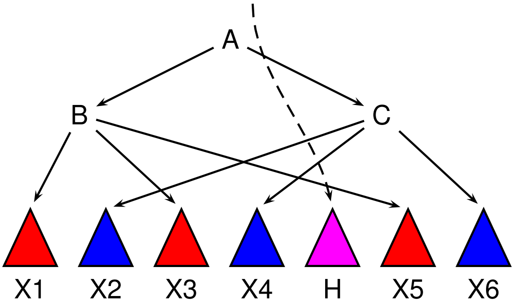

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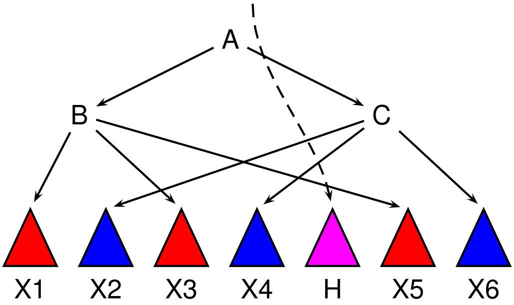

- **LFCRS**: $A \leftarrow f(B, C)$, f linear non erasing function on string tuples.

$$f(\langle x_1, x_3, x_5\rangle, \langle x_2, x_4, x_6\rangle) = \langle x_1 x_2 x_3 x_4, x_5 x_6\rangle$$

- **sRCG** $A(x_1.x_2.x_3.x_4, x_5.x_6) \leftarrow B(x_1, x_3, x_5), C(x_2, x_4, x_6)$
range variables x_i; concatenation "."; holes ","

Linear Context-Free Rewriting Systems (LCFRS) , a restricted form of *generalized CFGs*

A LCFRS is a tuple $G = (\mathcal{N}, \Sigma, S, \mathcal{P}, \mathcal{F})$ where

- $\mathcal{P}$ is a finite set of productions as follows, with $f \in \mathcal{F}$

$$A \leftarrow f(A_1, \ldots, A_n)$$

- $\mathcal{F}$ is a set of linear regular operations over tuples of strings in $\Sigma^\star$

$$f(\langle x_{1,1}, \ldots, x_{1,k_1} \rangle, \ldots, \langle x_{n,1}, \ldots, x_{1,k_n} \rangle) = \langle t_1, \ldots t_k \rangle$$

  where $V = \{x_{i,j}\}$ are variables (over $\Sigma^\star$) and $t_i \in (\Sigma \cup V)^\star$ and
  - (regular or non-erasing) $\forall x_{i,j}, \exists t_u, x_{i,j} \in t_u$
  - (linear) $\forall x_{i,j}, x_{i,j} \in t_u \wedge x_{i,j} \in t_v \Longrightarrow u = v$

Assuming $arity(S) = 1$,

$$L(G) = \{w | S \Longrightarrow \langle w \rangle\}$$

where

$$\begin{array}{ll} A \Longrightarrow f() & \text{if } A \rightarrow f() \in \mathcal{P} \\ A \Longrightarrow f(t_1, \ldots, t_n) & \text{if } A \rightarrow f(A_1, \ldots, A_n) \in \mathcal{P} \ \wedge \forall i,\ A_i \Longrightarrow t_i \end{array}$$

RCG

Range Concatenation Grammars (RCG) [Boullier] :
Constraints on intervals on the input string.
For language $a^n b^n c^n$

```
S(X @ Y @ Z) --> A(X,Y,Z).
A("a" @ X, "b" @ Y, "c" @ Z) -->
                 A(X,Y,Z).
A("","","") --> .
```

aabbcc	$S(]0,6]) \longrightarrow$
aabbcc	$A(]0,2],]2,4],]4,6]) \longrightarrow$
aabbcc	$A(]1,2],]3,4],]5,6]) \longrightarrow$
aabbcc	$A(]2,2],]4,4],]6,6]) \longrightarrow$

RCG is an operational formalism for encoding linguistic formalisms where discontinuous constituents are used.

RCG allow modular grammar writing

concatenation G(X @ Y) --> G1(X),G2(Y).

union G(X) --> G1(X) | G2(X).

intersection G(X) --> G1(X),G2(X).

Linear non-erasing positive RCGs equivalent to LCFRS
Full RCGs are PTIME (equivalent to Datalog)

Parsing MCS

- MCS have theoretical polynomial complexity $O(n^u)$ depending upon
  - degree of discontinuity, (also fanout, arity)
  - degree of interleaving, (also rank)

- But no uniform framework to express parsing strategies and tabular algorithms
  - operational device: Deterministic Tree Walking Transducer (Weir), but no tabular algorithm
  - operational formalism sRCG with tabular algorithm (Boullier) but not for prefix-valid strategies

Notion of Thread Automata to model discontinuity and interleaving through the suspension/resume of threads.

Tree Walking Automata

TWA may be used to check properties of (binary) trees
(by accepting or rejecting them)

A (non-deterministic) TWA is a tuple $A = (\mathcal{Q}, \Sigma, I, F, R, \delta)$ where

- $\mathcal{Q}$ is a finite set of states
- Σ a finite set of node labels
- $I, F, R \subset \mathcal{Q}$ the initial, accepting, rejecting states
- δ the finite set of transitions in $\mathcal{Q} \times \Sigma \times \mathrm{Pred} \times \mathrm{Dirs} \times \mathcal{Q}$ where
  - $\mathrm{Pred} \subset \{\mathrm{root}, \mathrm{left}, \mathrm{right}, \texttt{leaf}\}$ is a set of predicates for testing nodes
  - $\mathrm{Dirs} \subset \{\mathrm{stay}, \mathrm{up}, \mathrm{left}, \mathrm{right}\}$ a set of directions

  Deterministic TWA: $\delta : \mathcal{Q} \times \Sigma \times \mathrm{Pred} \mapsto \mathrm{Dirs} \times \mathcal{Q}$

Given a Σ-tree $\tau = (V, E)$, a configuration is given by $(\nu, q) \in V \times \mathcal{Q}$

Extensions: Pebble Automata (Engelfriet)

Similar to TWA, but emits strings when walking over a tree (in some tree set)

A deterministic TWD (Weir) is a tuple $T = (\mathcal{Q}, G, \Sigma_O, q_I, F, \delta)$ where

- $G = (\mathcal{N}, \Sigma_I, S, \mathcal{P})$ is a CFG
- Σ_O a finite set of output symbols
- $\delta : \mathcal{Q} \times (\mathcal{N} \cup \Sigma_I \cup \{\epsilon\}) \mapsto \mathrm{Dirs} \times \mathcal{Q} \times \Sigma_O$
  with $\mathrm{Dirs} = \{\mathrm{stay}, \mathrm{up}, \mathrm{down_1}, \ldots, \mathrm{down_n}\}$

A transition step given by

$$(q, \gamma, \nu, w) \stackrel{\star}{\vdash} (q', \gamma, \nu', w.v) \text{ if } \begin{cases} (q, \mathrm{dir}) = \delta(q, \mathrm{label}(\nu)) \\ \nu' = \mathrm{dir}(\nu) \end{cases}$$

The language generated by T defined as

$$L(T) = \{w | (q_I, \gamma, r_\gamma, \epsilon) \stackrel{\star}{\vdash} (q_f, \gamma, \uparrow, w)\}$$

with $q_f \in F$, γ a derivation tree for G with root r_γ
and $\uparrow$ a virtual node parent of r_γ

Weir's result: $L(\mathrm{DTWD}) = \mathrm{LCFRL}$

Thread Automata

Idea: Associate a thread p per constituent and

- create a subthread $p.u$ for a sub-constituent [PUSH]
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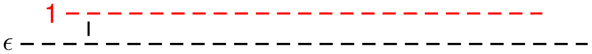

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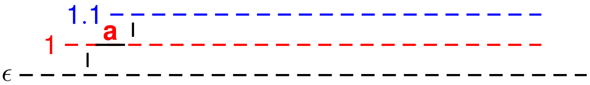

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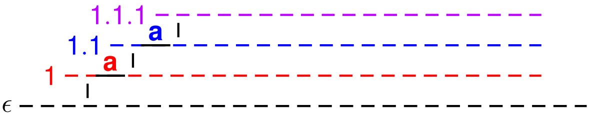

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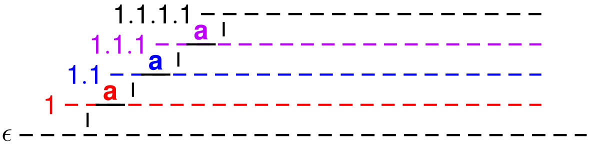

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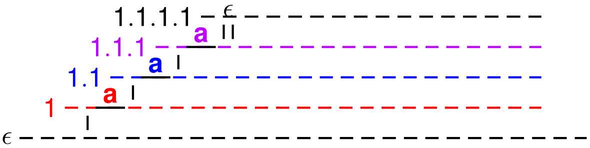

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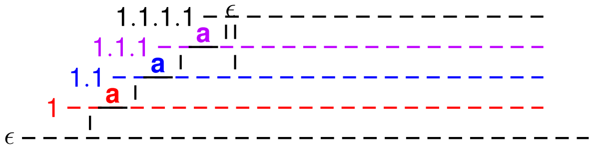

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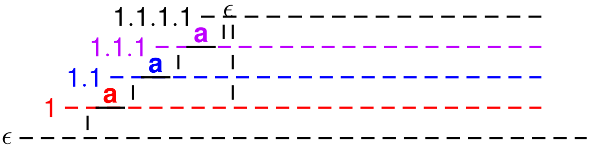

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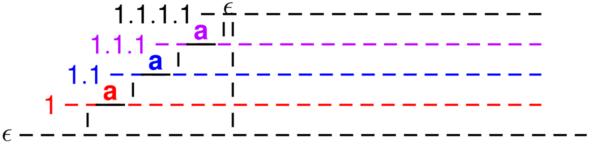

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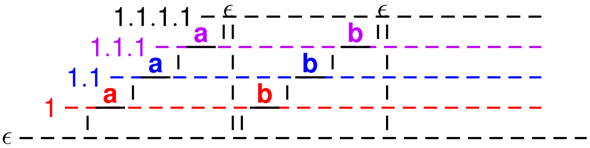

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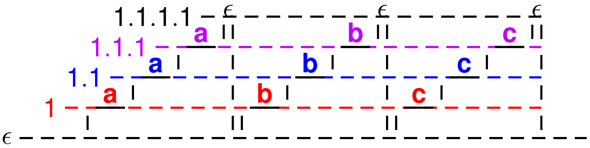

Formal presentation of TA

Configuration $\langle$position l, active thread path p, thread store $\mathcal{S} = \{p_i{:}A_i\}\rangle$
$\mathcal{S}$ closed by prefix: $p.u \in \mathrm{dom}(\mathcal{S}) \Longrightarrow p \in \mathrm{dom}(\mathcal{S})$
Note: stateless automata (but no problem for variants with states)

Triggering function $a = \Phi(A)$ amount of information needed to trigger transitions.
$\Longrightarrow$ useful to get linear compexity $O(|G|)$ w.r.t. grammar size $|G|$
Default: Φ = Identity

Driver function $u \in \delta(A)$ Drive thread creations and suspensions
$\Longrightarrow$ reduce number of transitions
(TA variants without δ should be possible)

Formal presentation of TA (cont'd)

SWAP $B \overset{\alpha}{\longmapsto} C$: Changes the content of the active thread, possibly scanning a terminal.

$$\langle l, p, \mathcal{S} \cup p{:}B \rangle \underset{\mathcal{T}}{\mid\!\!-\!\!-} \langle l + |\alpha|, p, \mathcal{S} \cup p{:}C \rangle \qquad a_l = \alpha \text{ if } \alpha \neq \epsilon$$

PUSH $b \longmapsto [b]C$: Creates a new subthread (unless present)

$$\langle l, p, \mathcal{S} \cup p{:}B \rangle \underset{\mathcal{T}}{\mid\!\!-\!\!-} \langle l, pu, \mathcal{S} \cup p{:}B \cup pu{:}C \rangle \quad (b, u) \in \Phi\delta(B) \wedge pu \notin \mathrm{do}$$

POP $[B]C \longmapsto D$: Terminates thread pu (if no existing subthreads).

$$\langle l, pu, \mathcal{S} \cup p{:}B \cup pu{:}C \rangle \underset{\mathcal{T}}{\mid\!\!-\!\!-} \langle l, p, \mathcal{S} \cup p{:}D \rangle \qquad pu \notin \mathrm{dom}(\mathcal{S})$$

SPUSH $b[C] \longmapsto [b]D$: Resumes the subthread pu (if already created)

$$\langle l, p, \mathcal{S} \cup p{:}B \cup pu{:}C \rangle \underset{\mathcal{T}}{\mid\!\!-\!\!-} \langle l, pu, \mathcal{S} \cup p{:}B \cup pu{:}D \rangle \quad (b, u^s) \in \Phi\delta(B)$$

SPOP $[B]c \longmapsto D[c]$: Resumes the parent thread p of pu

$$\langle l, pu, \mathcal{S} \cup p{:}B \cup pu{:}C \rangle \underset{\mathcal{T}}{\mid\!\!-\!\!-} \langle l, p, \mathcal{S} \cup p{:}D \cup pu{:}C \rangle \quad (c, \perp) \in \Phi\delta(C)$$

Characterizing Thread Automata

Key parameters:

- h maximal number of suspensions to the parent thread
  h finite ensures termination (of tabular parsing)
- d maximal number of simultaneously *alive* subthreads
- l maximal number of subthreads
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  $s \leq h + dh \leq h + lh$

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Worst-case Complexity:

$$\left.\begin{array}{l}\text{space } O(n^u)\\ \text{time } O(n^{1+u})\end{array}\right\} \text{where} \left\{\begin{array}{l} u = 2 + s + x \\ x = \min(s, (l-d)(h+1))\end{array}\right.$$

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Push-Down Automata (PDA) for CFG $\equiv$ TA(h=0,d=1,s=0)
$\Longrightarrow$ space $O(n^2)$ and time $O(n^3)$

Parsing TAGs

Idea: Assign a thread per elementary tree traversal (substitution or adjunction)
Suspend and return to parent thread to handle a foot node

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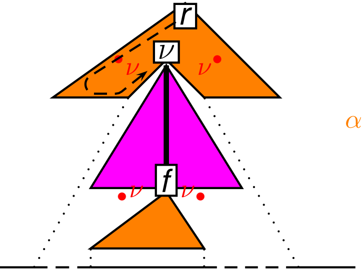

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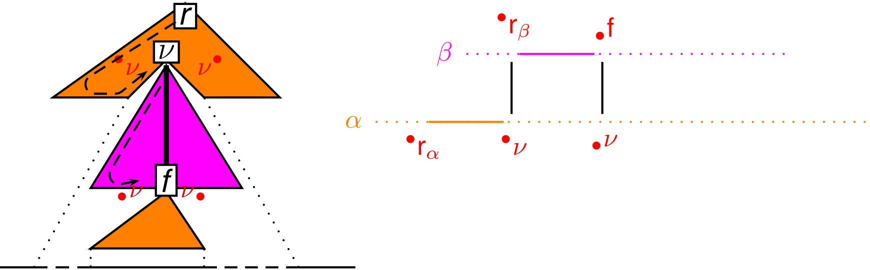

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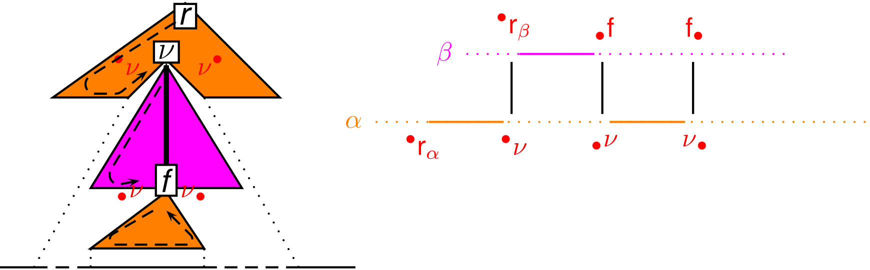

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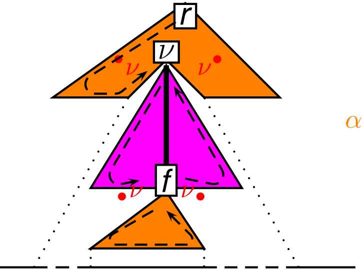

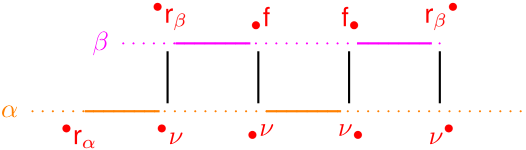

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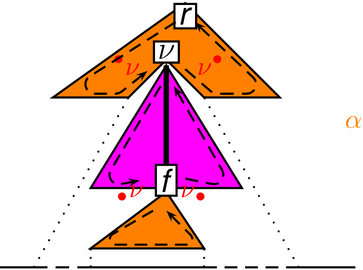

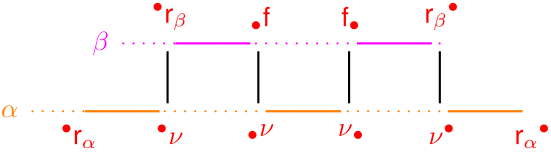

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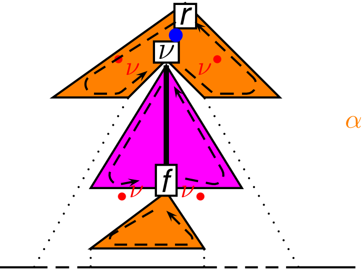

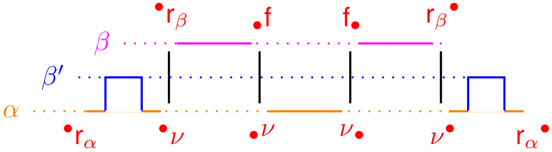

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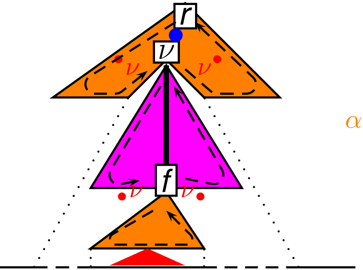

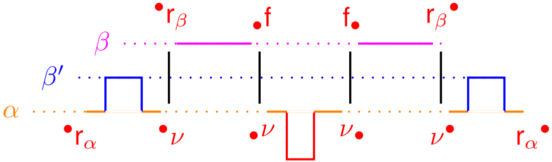

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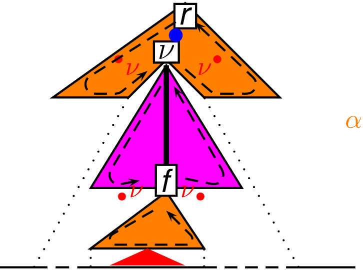

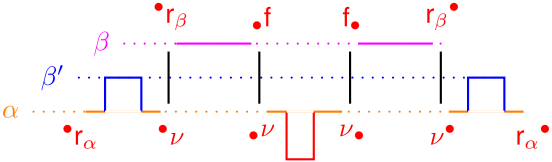

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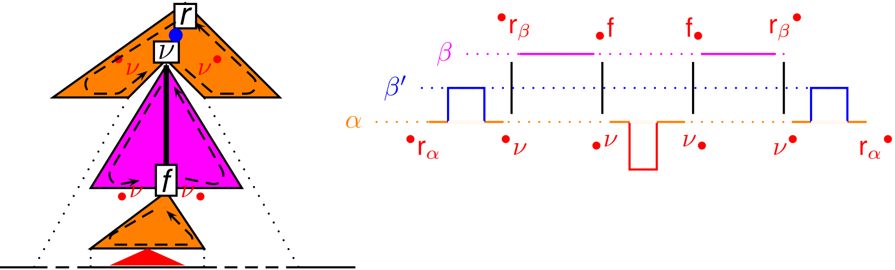
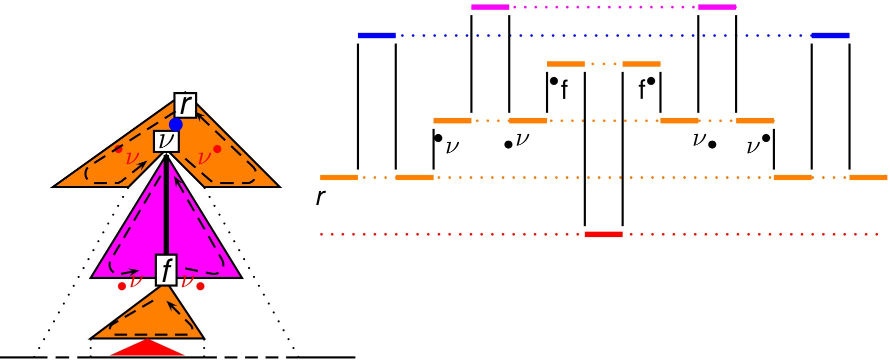


One thread per tree $h = 1, d = \max(\mathrm{depth}(\mathrm{trees}))$
$\Longrightarrow [s = 1 + d]$ space $O(n^{4+2d})$ and time $O(n^{5+2d})$

Parsing TAG: an alternate parsing strategy

Using more than one thread per elementary tree: 1 thread per subtree ($\sim$ LIG)
$\Longrightarrow$ implicit extraction of subtrees
$\Longrightarrow$ implicit normal form (using a third kind of tree operation)
$\Longrightarrow$ usual n^6 time complexity



Note: Similar to a TAG encoding in RCG proposed by Boullier

Using less threads

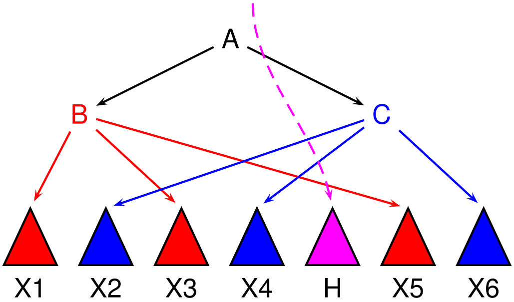
Always possible to reduce the number of live subthreads (down to 2).

- if a thread p has $d + 1$ subthreads, add a new subthread $p.v$ that inherits d subthreads of p
- generally increases the number of parent suspensions h
- but may also exploit good topological properties, such as well-nesting (TAGs).

Range Concatenation Grammars (Boullier)

$\gamma: \; A(X_1 X_2 X_3 X_4, X_5 X_6) \longrightarrow B(X_1, X_3, X_5) C(X_2, X_4, X_6)$

Ordered simple RCGs $\equiv$ Linear Context-Free Rewriting Systems (LCFRS)



Parsing (ordered simple) RCG

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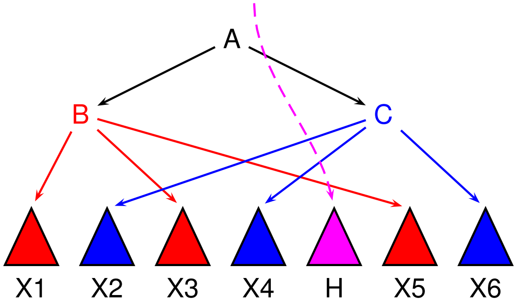

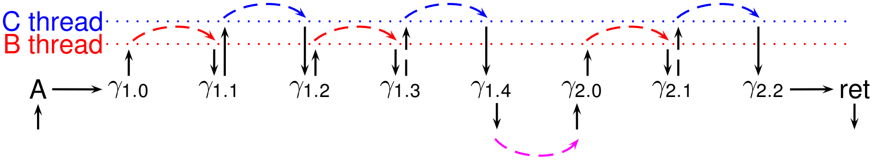

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$$\begin{cases} \rho \text{ stack of dotted nodes of trees being traversed} \\ \sigma \text{ sequence of root nodes of trees already traversed} \end{cases}$$

Parsing (set-local) MC-TAGs

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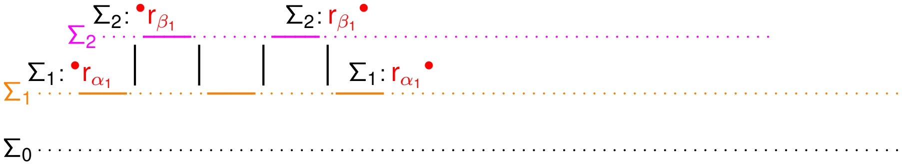

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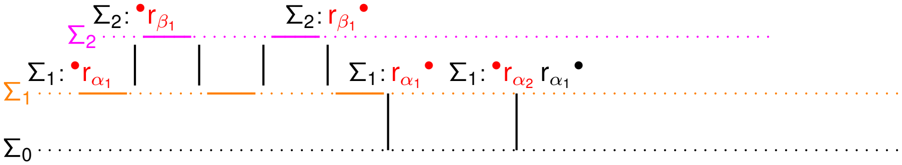

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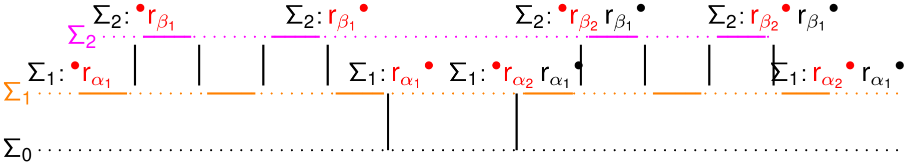

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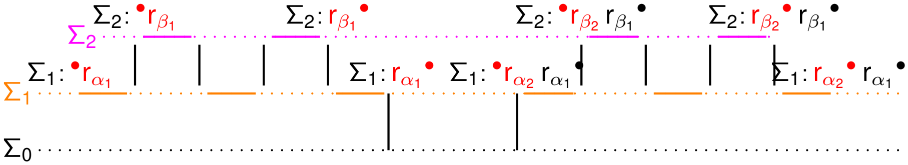

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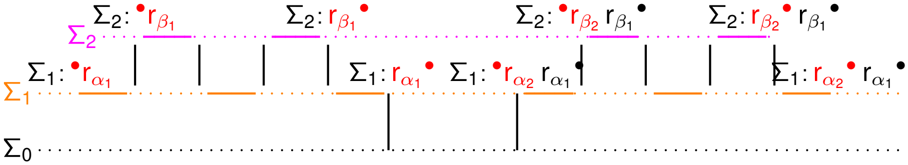

Time complexity $O(n^{3+2(m+v)})$ where $\begin{cases} m \text{ max number of trees per set} \\ v \text{ max number of nodes per set} \end{cases}$

Outline

Dynamic Programming interpretation

Direct evaluation of TA $\leadsto$ exponential complexity and non-termination

Use tabular techniques based on Dynamic Programming interpretation of TAs:

Principle: Identification of a class of subderivations that

- may be tabulated as compact items, removing non-pertinent information
- may be combined together and with transitions to retrieve all derivations

Methodology followed for PDAs (CFGs) and 2SAs (TAGs)

Dynamic Programming – Items

DP interpretation of TA derivations:
(Tabulated) Item $\equiv$ pertinent information about an (active) thread

1– Start point	3– (current) Parent suspensions
2– (current) End point	4– (current) Subthread suspensions for **live** subthreads

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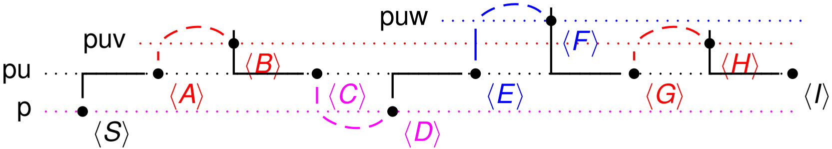

$\Longrightarrow$ Item: $\langle s\rangle / v : \langle a\rangle\langle b\rangle, \perp : \langle c\rangle\langle d\rangle, w : \langle e\rangle\langle f\rangle, v : \langle g\rangle\langle h\rangle / \langle I\rangle$

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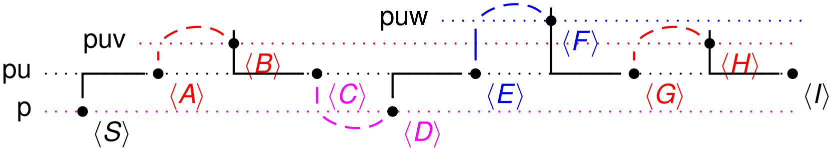

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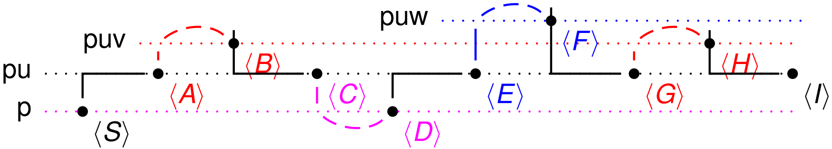

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Space complexity:

- at most 2 indices per suspensions + start + end = $2(1+s) \leq 2(1+h+dh)$
- Scanning parts generally of fixed length (independent of n)
  $\Longrightarrow$ 1 index per suspension

Dynamic Programming – Application rules

Based on following model:

$$\frac{\text{parent item} \quad \textcolor{red}{\text{son item}} \quad \textcolor{magenta}{\text{trans}}}{\text{parent or son extension}} \quad \{\text{fitting son and parent items}\}$$

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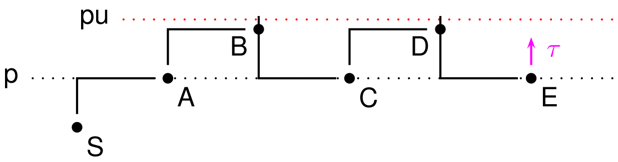

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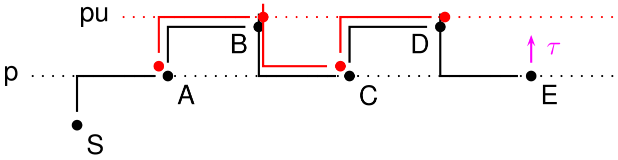

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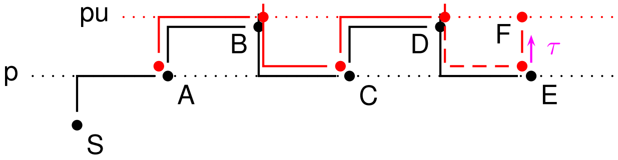

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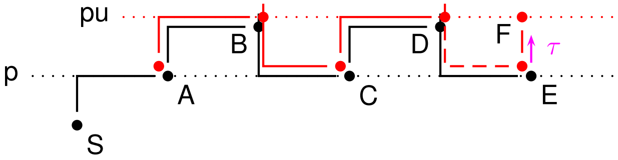

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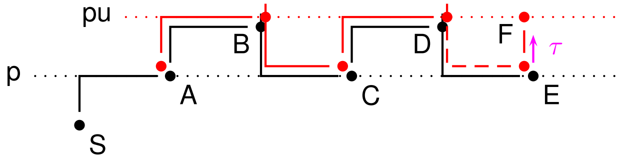

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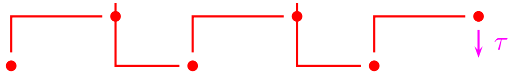

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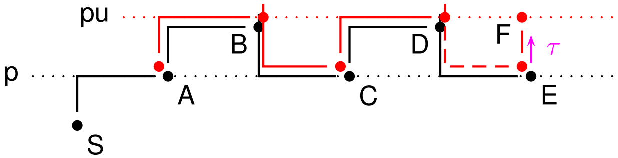

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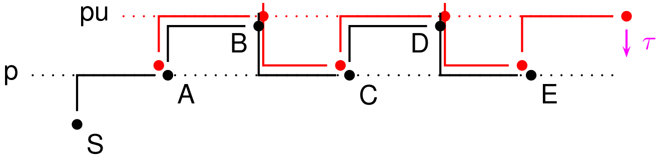

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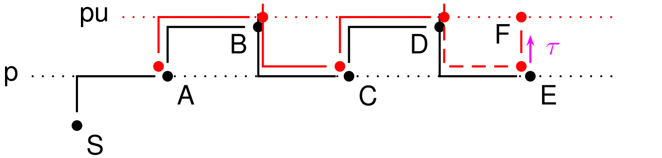

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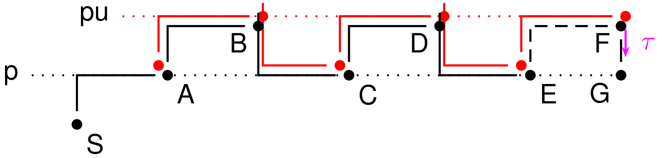

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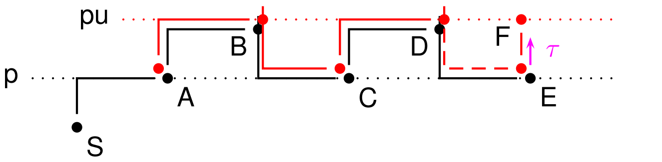
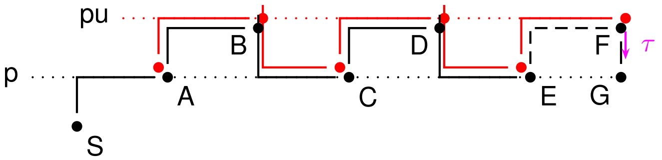


Case [SPOP]: son item up-extends parent item

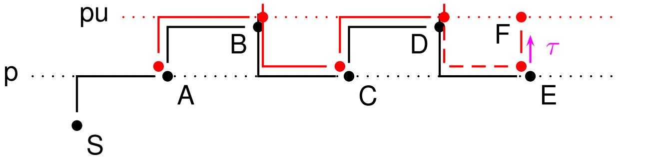


Dynamic Programming – Application rules

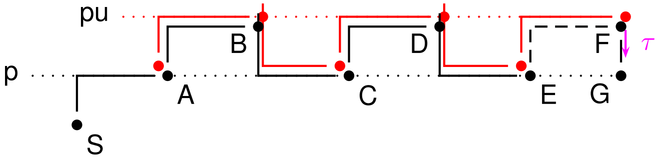
Based on following model:

$$\frac{\text{parent item} \quad \text{son item} \quad \text{trans}}{\text{parent or son extension}} \qquad \{\text{fitting son and parent items}\}$$

Case [SPUSH]: parent item down-extends son item



Case [SPOP]: son item up-extends parent item



Time complexity: all indices of parent item + end position of son item
ignore indices of son item not related to parent suspensions

Dynamic Programming: Rules

$$\frac{B \overset{\alpha}{\longmapsto} C \quad \langle a\rangle/\mathcal{S}/\langle B\rangle}{\langle a\rangle/\mathcal{S}/\langle C\rangle} \qquad a_r = \alpha \text{ if } \alpha \neq \epsilon \qquad \text{(SWAP)}$$

$$\frac{b \longmapsto [b]C \quad \star/\star/\langle B\rangle^I}{\langle b\rangle//\langle C\rangle} \qquad \{\ (b,u) \in \Phi\delta(B) \wedge u \notin \mathrm{ind}(I) \qquad \text{(PUSH)}$$

$$\frac{[B]C \longmapsto D \quad \langle a\rangle/\mathcal{S}/\langle B\rangle^I \quad J}{\langle a\rangle/\mathcal{S}_{/u}/\langle D\rangle} \qquad \begin{cases} J \nearrow^{u} I \wedge (b,u) \in \Phi\delta(B) \\ J^\bullet = \langle C\rangle \wedge \mathrm{ind}(J) \subset \{\perp\} \end{cases} \qquad \text{(POP)}$$

$$\frac{b[C] \longmapsto [b]D \quad I \quad \langle a\rangle/\mathcal{S}/\langle C\rangle^J}{\langle a\rangle/\mathcal{S},\perp:\langle c\rangle\langle b\rangle//\langle D\rangle} \qquad \begin{cases} I \searrow_{u} J \wedge I^\bullet = \langle B\rangle \\ (b,u) \in \Phi\delta(B) \wedge (c,\perp) \in \Phi\delta(C) \end{cases} \qquad \text{(SPUSH)}$$

$$\frac{[B]c \longmapsto D[c] \quad \langle a\rangle/\mathcal{S}/\langle B\rangle^I \quad J}{\langle a\rangle/\mathcal{S},u:\langle b\rangle\langle c\rangle//\langle D\rangle} \qquad \begin{cases} J \nearrow^{u} I \wedge (b,u) \in \Phi\delta(B) \\ J^\bullet = \langle C\rangle \wedge (c,\perp) \in \Phi\delta(C) \end{cases} \qquad \text{(SPOP)}$$