TD 9: Pushdown Systems

Exercise 1 (Regular Valuations). The course notes prove the decidability of LTL model checking with simple valuations \( \nu : P \times \Gamma^* \rightarrow 2^{AP} \) satisfying \( \nu(qZ\gamma) = \nu(qZ) \) for all \( q \) in \( P \), \( Z \) in \( \Gamma \) and \( \gamma \) in \( \Gamma^* \).

A regular valuation is defined through a collection of finite complete deterministic automata \( A_p = (Q_p, \Gamma \sqcup P, \delta_p, q_{0,p}, F_p) \) for each \( p \) in \( AP \), s.t.
\[
\nu(qZ\gamma) = \{ p \in AP \mid \delta_p(q_{0,p}, \gamma^RZq) \in F_p \}
\]
i.e. each \( A_p \) is run bottom-to-top on the stack and pushdown state, and \( p \) holds if we reach a final state in \( F_p \).

Show that the LTL model-checking problem with regular valuations for PDS can be reduced to the LTL model-checking problem with simple valuations.

Exercise 2 (CTL∗ Model Checking). Show that CTL∗ model checking with regular valuations can be reduced to LTL model checking with simple valuations.

Exercise 3 (ExpTime-Hardness for LTL with Regular Valuations). Show that the model-checking problem for PDS and LTL formulae with regular valuations is ExpTime-hard.

Exercise 4 (Multi-Pushdown Systems). A n-dimensional multi-pushdown system is a tuple \( M = (P, \Gamma, (\Delta_i)_{0<i \leq n}) \) where \( n \geq 1 \) is the number of stacks, \( P \) a finite set of states, \( \Gamma \) a finite stack alphabet, and each \( \Delta_i \subseteq P \times \Gamma \times P \times \Gamma^* \) is a finite transition relation. A configuration of a n-MPDS is a tuple \( c = (q, \gamma_1, \ldots, \gamma_n) \) in \( P \times (\Gamma^*)^n \). The transition relation \( \Rightarrow \) on configurations is defined as \( \Rightarrow = \bigcup_{0<i \leq n} \Rightarrow_i \), where
\[
(q, \gamma_1, \ldots, Z\gamma_i, \ldots, \gamma_n) \Rightarrow_i (q', \gamma_1, \ldots, \gamma'_i\gamma_i, \ldots, \gamma_n)
\]
iff \( qZ \xrightarrow{i} q'\gamma'_i \) is in \( \Delta_i \).

1. Show that the control state reachability problem, i.e. given an initial configuration \( c \) in \( P \times \Gamma^n \) and a control state \( p \in P \), whether there exist \( \gamma_1, \ldots, \gamma_n \) s.t. \( c \Rightarrow^* (q, \gamma_1, \ldots, \gamma_n) \), is undecidable as soon as \( n \geq 2 \).

2. Let us consider a restriction on \( \Rightarrow^* \): \( k \)-bounded runs are defined as the \( k \)th iterates \( c \rightarrow^k c' \) of the relation
\[
c \rightarrow c' \text{ iff } \exists i. c \Rightarrow_i^k c'
\]
i.e. a $k$-bounded run can be decomposed into $k$ subruns where a single PDS is running.

Show that the $k$-bounded control-state reachability problem, i.e. given an initial configuration $c$ in $P \times \Gamma^n$ and a control state $p \in P$, whether there exist $\gamma_1, \ldots, \gamma_n$ s.t. $c \rightarrow^k (q, \gamma_1, \ldots, \gamma_n)$, is decidable.