TD 9: Pushdown Systems

Exercise 1 (Regular Valuations)**.** The course notes prove the decidability of LTL model checking with *simple valuations* $\nu : P \times \Gamma^* \to 2^{\mathrm{AP}}$ satisfying $\nu(qZ\gamma) = \nu(qZ)$ for all q in P, Z in Γ and γ in Γ^*.

A *regular valuation* is defined through a collection of finite complete deterministic automata $\mathcal{A}_p = \langle Q_p, \Gamma \uplus P, \delta_p, q_{0,p}, F_p \rangle$ for each p in AP, s.t.

$$\nu(qZ\gamma) = \{p \in \mathrm{AP} \mid \delta_p(q_{0,p}, \gamma^R Z q) \in F_p\} ,$$

i.e. each $\mathcal{A}_p$ is run bottom-to-top on the stack and pushdown state, and p holds if we reach a final state in F_p.

Show that the LTL model-checking problem with regular valuations for PDS can be reduced to the LTL model-checking problem with simple valuations.

Exercise 2 (CTL* Model Checking)**.** Show that CTL* model checking with regular valuations can be reduced to LTL model checking with simple valuations.

Exercise 3 (ExpTime-Hardness for LTL with Regular Valuations)**.** Show that the model-checking problem for PDS and LTL formulæ with regular valuations is ExpTime-hard.

Exercise 4 (Multi-Pushdown Systems)**.** A n-dimensional *multi-pushdown system* is a tuple $\mathcal{M} = \langle P, \Gamma, (\Delta_i)_{0<i\leq n} \rangle$ where $n \geq 1$ is the number of stacks, P a finite set of states, Γ a finite stack alphabet, and each $\Delta_i \subseteq P \times \Gamma \times P \times \Gamma^*$ is a finite transition relation. A configuration of a n-MPDS is a tuple $c = (q, \gamma_1, \ldots, \gamma_n)$ in $P \times (\Gamma^*)^n$. The *transition* relation $\Rightarrow$ on configurations is defined as $\Rightarrow = \bigcup_{0<i\leq n} \Rightarrow_i$, where

$$(q, \gamma_1, \ldots, Z\gamma_i, \ldots, \gamma_n) \Rightarrow_i (q', \gamma_1, \ldots, \gamma_i'\gamma_i, \ldots, \gamma_n)$$

iff $qZ \hookrightarrow_i q'\gamma_i'$ is in Δ_i.

1. Show that the *control state reachability problem*, i.e. given an initial configuration c in $P \times \Gamma^n$ and a control state $p \in P$, whether there exist $\gamma_1, \ldots, \gamma_n$ s.t. $c \Rightarrow^* (q, \gamma_1, \ldots, \gamma_n)$, is undecidable as soon as $n \geq 2$.

2. Let us consider a restriction on $\Rightarrow^*$: k-bounded runs are defined as the kth iterates $c \to^k c'$ of the relation

$$c \to c' \text{ iff } \exists i . c \Rightarrow_i^* c'$$

i.e. a k-bounded run can be decomposed into k subruns where a single PDS is running.

Show that the k-bounded control-state reachability problem, i.e. given an initial configuration c in $P \times \Gamma^n$ and a control state $p \in P$, whether there exist $\gamma_1, \ldots, \gamma_n$ s.t. $c \to^k (q, \gamma_1, \ldots, \gamma_n)$, is decidable.