

TD 5

1 Model Checking for LTL

Exercise 1 (Model Checking a Path). Consider the time flow \((\mathbb{N}, <)\). We want to verify a model which is an ultimately periodic word \(w = uv^\omega\) with \(u\) in \(\Sigma^*\) and \(v\) in \(\Sigma^+\).

Give an algorithm for checking whether \(w, 0 \models \varphi\) holds, where \(\varphi\) is a TL\((X, U')\) formula, in time bounded by \(O(|uv| \cdot |\varphi|)\).

Exercise 2 (Complexity of TL\((X)\)). We want to show that TL\((X)\) existential model checking is NP-complete (instead of PSPACE-complete for the full TL\((X, U')\)).

1. Show that MC\(^3\)\((X)\) is in NP.
2. Reduce 3SAT to MC\(^3\)\((X)\) in order to prove NP-hardness.

2 CTL*

Exercise 3 (Equivalences). Are the following formulæ equivalent?

1. AXAG\(\varphi\) and AXG\(\varphi\)
2. EXEG\(\varphi\) and EXG\(\varphi\)
3. A(\(\varphi \land \psi\)) and A\(\varphi\) \(\land\) A\(\psi\)
4. E(\(\varphi \land \psi\)) and E\(\varphi\) \(\land\) E\(\psi\)
5. \(\neg A(\varphi \Rightarrow \psi)\) and E(\(\varphi \land \neg \psi\))

Exercise 4 (Model Checking).
Check whether the above Kripke structure verifies the following CTL* formula:
\[ E( X(a \land \neg b) \land XA(b U (Ga))) \].

3 CTL and CTL+

Exercise 5 (CTL Equivalences).

1. Are the two formulæ \( \varphi = AG(EP) \) and \( \psi = EFp \) equivalent? Does one imply the other?

2. Same questions for \( \varphi = EGq \lor (EGp \land EFq) \) and \( \psi = E(p U q) \).

Exercise 6 (CTL+). CTL+ extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

\[
\begin{align*}
\text{f} & ::= \top | a | f \land g | \neg f | E \varphi | A \varphi & \text{(state formulæ \( f, g \))} \\
\text{\varphi} & ::= \varphi \land \psi | \neg \varphi | Xf | f U g & \text{(path formulæ \( \varphi, \psi \))}
\end{align*}
\]

where \( a \) is an atomic proposition. The associated semantics is that of CTL*.

We want to prove that, for any CTL+ formula, there exists an equivalent CTL formula.

1. Give an equivalent CTL formula for
\[ E((a_1 U b_1) \land (a_2 U b_2)) \].

2. Generalize your translation for any formula of form
\[ E( \bigwedge_{i=1,\ldots,n} (\psi_i U \psi_i') \land G\varphi) \]. \hspace{1cm} (1)

What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL+ formula:
\[ E(Xa \land (b U c)) \].

4. Using subformulæ of form (1) and EX modalities, give an equivalent CTL formula to
\[ E(X\varphi \land \bigwedge_{i=1,\ldots,n} (\psi_i U \psi_i') \land G\varphi') \]. \hspace{1cm} (2)

What is the complexity of your translation?
5. We only have to transform any $\text{CTL}^+$ formula into (nested) disjuncts of form (2). Detail this translation for the following formula:

$$A((Fa \lor Xa \lor X\neg b \lor F\neg d) \land (d \lor \neg c)).$$

**Exercise 7 (Fair CTL).** We consider *strong* fairness constraints, which are conjunctions of formulæ of form

$$\text{GF} \psi_1 \Rightarrow \text{GF} \psi_2.$$

We want to check whether the following Kripke structure fairly verifies

$$\varphi = \text{AGAF}a$$

under the fairness requirement $e$ defined by

$$\psi_1 = b \land \neg a$$
$$\psi_2 = E(b \lor (a \land \neg b))$$
$$e = \text{GF} \psi_1 \Rightarrow \text{GF} \psi_2.$$

1. Compute $[[\psi_1]]_e$ and $[[\psi_2]]_e$.
2. Compute $[[\text{EG} \top]]_e$.
3. Compute $[[\varphi]]_e$. 
