TD 5: CTL* and CTL

1 CTL*

Exercise 1 (Equivalences). Are the following formulæ equivalent?

1. AXAGφ and AXGφ
2. EXEGφ and EXGφ
3. A(φ ∧ ψ) and Aφ ∧ Aψ
4. E(φ ∧ ψ) and Eφ ∧ Eψ
5. ¬A(φ ⇒ ψ) and E(φ ∧ ¬ψ)

Exercise 2 (Model Checking).

Check whether the above Kripke structure verifies the following CTL* formula:

\[ E(X(a ∧ ¬b) ∧ AX(b U (Ga))) \].

2 CTL and CTL+

Exercise 3 (CTL Equivalences).

1. Are the two formulæ \( φ = AG(EFp) \) and \( ψ = EFp \) equivalent? Does one imply the other?

2. Same questions for \( φ = EGq ∨ (EGp ∧ EFq) \) and \( ψ = E(p U q) \).
Exercise 4 (CTL\(^+\)). CTL\(^+\) extends CTL by allowing boolean connectives on path formulæ, according to the following abstract syntax:

\[
\begin{align*}
    f & ::= \top \mid a \mid f \land g \mid \neg f \mid E\varphi \mid A\varphi \\
    \varphi & ::= \varphi \land \psi \mid \neg \varphi \mid Xf \mid f \lor g
\end{align*}
\]

where \(a\) is an atomic proposition. The associated semantics is that of CTL\(^*\).

We want to prove that, for any CTL\(^+\) formula, there exists an equivalent CTL formula.

1. Give an equivalent CTL formula for
   \(E((a_1 \lor b_1) \land (a_2 \lor b_2))\).

2. Generalize your translation for any formula of form
   \(E(\bigwedge_{i=1}^{n}(\psi_i \lor \psi_i') \land G\varphi)\). \tag{1}

   What is the complexity of your translation?

3. Give an equivalent CTL formula for the following CTL\(^+\) formula:
   \(E(Xa \land (b \lor c))\).

4. Using subformulæ of form (1) and EX modalities, give an equivalent CTL formula to
   \(E(X\varphi \land \bigwedge_{i=1}^{n}(\psi_i \lor \psi_i') \land G\varphi')\). \tag{2}

   What is the complexity of your translation?

5. We only have to transform any CTL\(^+\) formula into (nested) disjuncts of form (2).
   Detail this translation for the following formula:
   \(A((F \lor Xa \lor X\neg b \lor F\neg d) \land (d \lor \neg e))\).

Exercise 5 (Fair CTL). We consider strong fairness constraints, which are conjunctions of formulæ of form

\[GF\psi_1 \Rightarrow GF\psi_2.\]

We want to check whether the following Kripke structure fairly verifies

\[\varphi = AGAFAa\]

under the fairness requirement \(e\) defined by

\[
\begin{align*}
    \psi_1 &= b \land \neg a \\
    \psi_2 &= E(b \lor (a \land \neg b)) \\
    e &= GF\psi_1 \Rightarrow GF\psi_2.
\end{align*}
\]
1. Compute $[\psi_1]$ and $[\psi_2]$.

2. Compute $[EG\top]_e$.

3. Compute $[\varphi]_e$. 