Here is a set of simple exercises in preparation for the mid-semester exam. The first two exercises were in last year’s exam. There ought to be some time left for discussing the home assignment.

**Exercise 1** (LTL and Automata). In the following, AP is a finite set of atomic propositions and \( \Sigma = 2^{\text{AP}} \) is the corresponding finite alphabet. Let \( \varphi \) be a LTL\((\text{AP, } X, U, Y, S)\) a formula. We define
\[
L(G \varphi) = \{ \sigma \in \Sigma^\omega \mid \forall n \in \mathbb{N}, \sigma, n \models \varphi \}.
\]
Consider the following formulae:
\[
\begin{align*}
\varphi_1 &= (\neg \text{lock} S \text{unlock}) \Rightarrow (\neg \text{use} W \text{lock}) \\
\varphi_2 &= (\neg \text{lock} S \text{unlock}) \Rightarrow (\neg \text{use} \lor \text{lock}) \\
\varphi_3 &= \text{unlock} \Rightarrow (\neg \text{use} W \text{lock})
\end{align*}
\]
on the set of atomic propositions \( \text{AP} = \{ \text{lock, unlock, use} \} \). For each \( 1 \leq i \leq 3 \), we denote \( L(G \varphi_i) \) by \( L_i \).

1. Show that \( L_1 = L_2 \).
2. Show that \( L_1 \subseteq L_3 \) and \( L_1 \neq L_3 \).
3. Give a pure future formula \( \varphi_4 \) in LTL\((\text{AP, } X, U)\) such that \( L(G \varphi_4) = L_1 \). Of course, the equality has to be demonstrated.
4. Construct, using the method described in the lecture notes, the generalized Büchi automaton \( A_{G \varphi_3} \) that recognizes \( L_3 \). To make things easier, we note \( p = \text{unlock} \), \( q = \text{lock} \), and \( r = \text{use} \) such that \( \varphi_3 = \neg p \lor (G \neg r) \lor (\neg r U q) \).
5. Show how to obtain a deterministic Büchi automaton with all its states final for \( L_3 \).

**Exercise 2** (CTL and CTL*). Let us consider the following model \( M \):

![Model Diagram]

\[
\begin{align*}
2 & \xrightarrow{0} \emptyset \\
4 & \xrightarrow{p} \{ p \} \\
6 & \xrightarrow{0} \emptyset \\
1 & \xrightarrow{p,q} \{ p,q \} \\
3 & \xrightarrow{q} \{ q \} \\
5 & \xrightarrow{q} \{ q \}
\end{align*}
\]
Let us recall that, if \( \varphi \) is a state formula, then \( \llbracket \varphi \rrbracket \) denotes the set of states of \( M \) that verify \( \varphi \).

1. Compute \( \llbracket EGFp \rrbracket \).
2. Compute \( \llbracket AGFq \rrbracket \).
3. Compute \( \llbracket \varphi_1 \rrbracket \) where \( \varphi_1 = Eq \cup (p \land \neg q) \).
4. Compute \( \llbracket \varphi_2 \rrbracket \) where \( \varphi_2 = A(GFp \land (GF(\neg p \land q) \land FG\neg \varphi_1)) \).
5. For each state \( i \) of \( M \), provide a CTL formula \( \xi_i \) such that \( \llbracket \xi_i \rrbracket = \{ i \} \).

**Exercise 3** (Complexity of LTL(\( X \))). We want to show that LTL(\( X \)) existential model checking is NP-complete (instead of PSPACE-complete for the full LTL(\( X, U \))).

1. Show that \( MC^3(X) \) is in NP.
2. Reduce 3SAT to \( MC^3(X) \) in order to prove NP-hardness.