TD 5

Here is a set of simple exercises in preparation for the mid-semester exam. The first two exercises were in last year's exam. There ought to be some time left for discussing the home assignment.

Exercise 1 (LTL and Automata)**.** In the following, AP is a finite set of atomic propositions and $\Sigma = 2^{\mathrm{AP}}$ is the corresponding finite alphabet. Let φ be a LTL(AP, $\mathsf{X}, \mathsf{U}, \mathsf{Y}, \mathsf{S}$) a formula. We define

$$L(\mathsf{G}\varphi) = \{\sigma \in \Sigma^\omega \mid \forall n \in \mathbb{N}, \sigma, n \models \varphi\} .$$

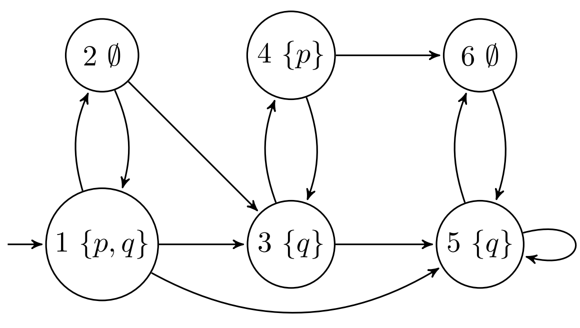
Consider the following formulæ:

$$\begin{aligned}
\varphi_1 &= (\neg\text{lock } \mathsf{S} \text{ unlock}) \Rightarrow (\neg\text{use } \mathsf{W} \text{ lock})\\
\varphi_2 &= (\neg\text{lock } \mathsf{S} \text{ unlock}) \Rightarrow (\neg\text{use} \vee \text{lock})\\
\varphi_3 &= \text{unlock} \Rightarrow (\neg\text{use } \mathsf{W} \text{ lock})
\end{aligned}$$

on the set of atomic propositions AP = {lock, unlock, use}. For each $1 \leq i \leq 3$, we denote $L(\mathsf{G}\varphi_i)$ by L_i.

1. Show that $L_1 = L_2$.
2. Show that $L_1 \subseteq L_3$ and $L_1 \neq L_3$.
3. Give a *pure future* formula φ_4 in LTL(AP, X, U) such that $L(\mathsf{G}\varphi_4) = L_1$. Of course, the equality has to be demonstrated.
4. Construct, using the method described in the lecture notes, the generalized Büchi automaton $\mathcal{A}_{\mathsf{G}\varphi_3}$ that recognizes L_3. To make things easier, we note $p =$ unlock, $q =$ lock, and $r =$ use such that $\varphi_3 = \neg p \vee (\mathsf{G}\neg r) \vee (\neg r \mathsf{U} q)$.
5. Show how to obtain a deterministic Büchi automaton with all its states final for L_3.

Exercise 2 (CTL and CTL*)**.** Let us consider the following model M:



Let us recall that, if φ is a state formula, then $[\![\varphi]\!]$ denotes the set of states of M that verify φ.

1. Compute $[\![\mathsf{EGF}p]\!]$.
2. Compute $[\![\mathsf{AGF}q]\!]$.
3. Compute $[\![\varphi_1]\!]$ where $\varphi_1 = \mathsf{E}q\ \mathsf{U}\ (p \wedge \neg q)$.
4. Compute $[\![\varphi_2]\!]$ where $\varphi_2 = \mathsf{A}(\mathsf{GF}p \wedge (\mathsf{GF}(\neg p \wedge q) \wedge \mathsf{FG}\neg\varphi_1))$.
5. For each state i of M, provide a CTL formula ξ_i such that $[\![\xi_i]\!] = \{i\}$.

Exercise 3 (Complexity of LTL(X))**.** We want to show that LTL(X) existential model checking is NP-complete (instead of PSPACE-complete for the full LTL(X, U)).

1. Show that $\mathsf{MC}^{\exists}(\mathsf{X})$ is in NP.
2. Reduce 3SAT to $\mathsf{MC}^{\exists}(\mathsf{X})$ in order to prove NP-hardness.