Alternating Vector Addition Systems with States

S. Schmitz, based on joint works with J.-B. Courtois, M. Jurdziński, and R. Lazić

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Séminaire MV, LaBRI, Dec. 4th 2014

Outline

alternating VASS and asymmetric vector addition games

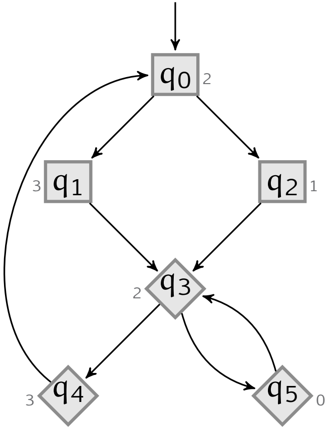
applications

- substructural logics
- regular simulations
- energy games

complexity

Recap: Parity Games

- two Players $\Diamond$ and $\Box$: partitioned state space $Q = Q_\Diamond \uplus Q_\Box$
- colour in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest colour seen infinitely often is even



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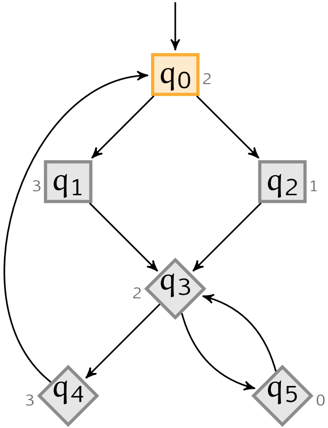

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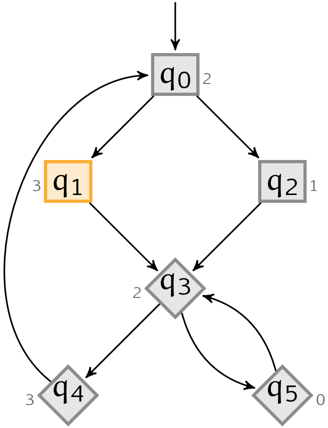

Recap: Parity Games

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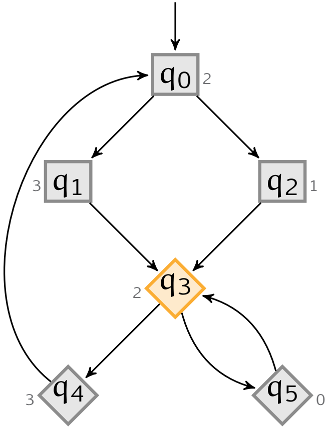

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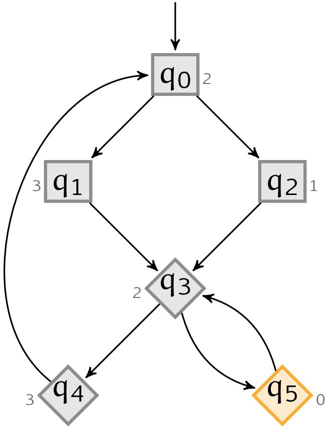

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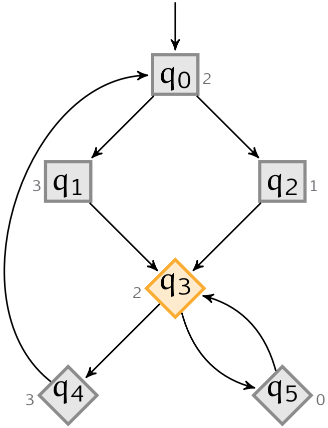

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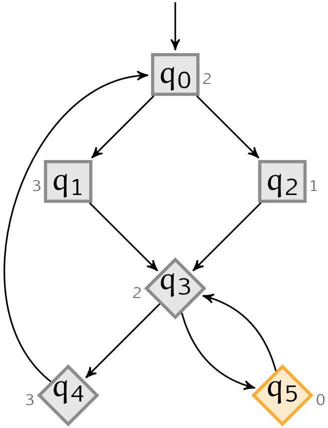

Vector Addition Games

- two Players $\Diamond$ and $\Box$: partitioned state space $Q = Q_\Diamond \uplus Q_\Box$
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- defines an infinite arena in $Q \times \mathbb{N}^d$
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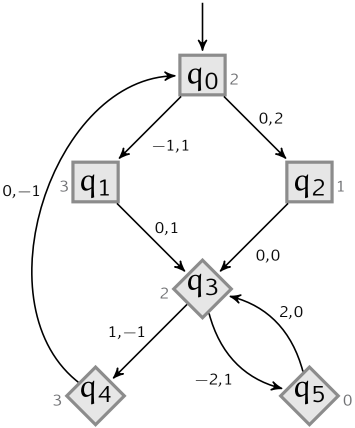

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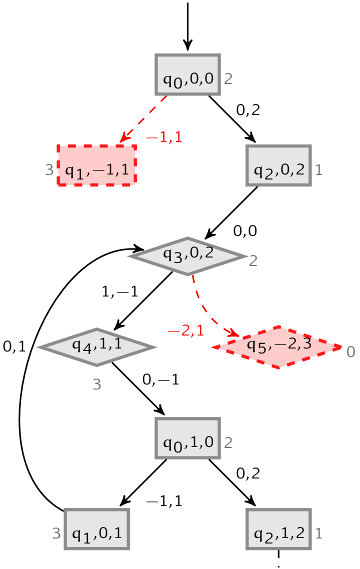

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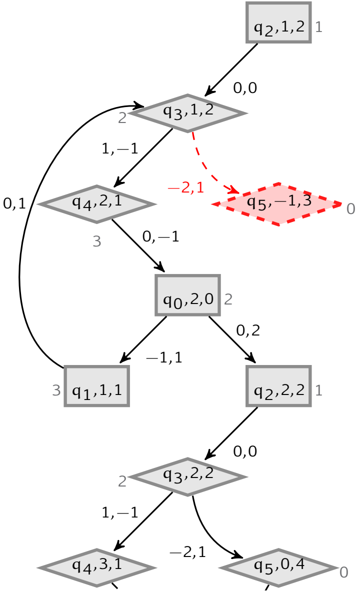

Game Objectives

Monotone objectives:

coverability given $q_\ell \in Q$, $\diamond$ wins if any configuration in $\{q_\ell\} \times \mathbb{N}^d$ is visited

non-termination $\diamond$ wins if the play is infinite

parity given a colouring $c: Q \to \{1, \ldots, k\}$, $\diamond$ wins if the least colour seen infinitely often is even

Non-monotone objective:

reachability given $q_\ell \in Q$, $\diamond$ wins if the configuration $(q_\ell, \mathbf{0})$ is visited

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Initial Credit

Given $q_r \in Q$:

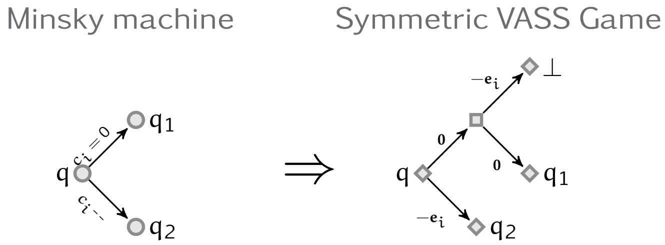
fixed start from the configuration $(q_r, \mathbf{0})$

unknown $\Diamond$ chooses an initial vector $\mathbf{v}_r \in \mathbb{N}^d$
start from the configuration $(q_r, \mathbf{v}_r)$

Coverability VASS Games

(Raskin, Samuelides, and Van Begin, 2005)

Player □ can enforce zero-tests:



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Coverability VASS games with fixed initial credit are undecidable.

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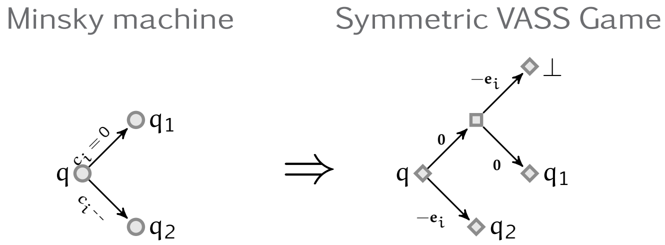

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Asymmetric VASS Games

aka. vector games (Kanovich, 1995), B-games (Raskin et al., 2005), single-sided games (Abdulla et al., 2013)

- $Q = Q_\Diamond \uplus Q_\Box$, resp. Controller and Environment
- $T_\Diamond \subseteq Q_\Diamond \times \mathbb{Z}^d \times Q$:

$$q_\Diamond \xrightarrow{\mathbf{u}} q'$$

- $T_\Box \subseteq Q_\Box \times \{\mathbf{0}\} \times Q$:

$$\begin{tikzcd}[row sep=small] & q_1 \\ q_\Box \arrow[ur] \arrow[dr] & \\ & q_2 \end{tikzcd}$$

Alternating VASS

aka. "and-branching" (Lincoln et al., 1992; Urquhart, 1999)

Q finite set of states

q_r initial state in Q

T_u finite set of unary transitions $\subseteq Q \times \mathbb{Z}^d \times Q$:

$$q \xrightarrow{\mathbf{u}} q'$$

T_f set of fork transitions $\subseteq Q^3$:

$$\begin{array}{ccc} & & q_1 \\ & \nearrow & \\ q & & \\ & \searrow & \\ & & q_2 \end{array}$$

Tree Semantics $\cong$ Controller Strategies

run in $T(Q \times \mathbb{N}^d)$:

Initial

$$q_r, \mathbf{0}$$

unary rule in T_u

$$\begin{tikzcd} q, \mathbf{v} \arrow[d, "\mathbf{u}"] \\ q', \mathbf{v} + \mathbf{u} \end{tikzcd}$$

blocking transition: $\mathbf{v} + \mathbf{u} \geqslant \mathbf{0}$

fork rule in T_f

$$\begin{tikzcd} & q, \mathbf{v} \arrow[dl] \arrow[dr] & \\ q_1, \mathbf{v} & & q_2, \mathbf{v} \end{tikzcd}$$

Monotone Games

Lemma

If Controller wins a monotone AVASS game from some configuration $(q,\mathbf{v})$ and $\mathbf{v}' \geqslant \mathbf{v}$, then he also wins from $(q,\mathbf{v}')$.

Corollary (using Dickson's Lemma)

- *finite-memory strategies suffice for Controller*
- *coverability and non-termination AVASS games are decidable*
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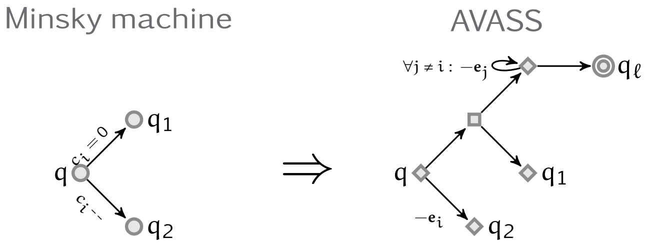
Reachability Objective (1/2)

(Lincoln, Mitchell, Scedrov, and Shankar, 1992)

Player □ can enforce zero-tests using the reachability objective $(q_\ell, \mathbf{0})$:



Theorem (Lincoln et al., 1992)

Reachability AVASS games with fixed initial credit are undecidable.

REACHABILITY OBJECTIVE (2/2)

(Urquhart, 1999)

Unknown initial credit ≅ gainy game
where $\forall q \in Q. \forall 1 \leqslant i \leqslant d. q \xrightarrow{\mathbf{e}_i} q \in T_u$

Theorem (Urquhart, 1999; Lazić and S., 2014)
Reachability AVASS games with unknown initial credit are ACKERMANN*-complete.*

Complexity Preview

	initial credit	
objective	fixed	unknown
coverability	2EXP (Courtois and S., 2014)	P (trivial)
non-termination	2EXP $\leqslant$? $\leqslant$ TOWER (Brázdil et al., 2010)	coNP (Chatterjee et al., 2010)
parity	2EXP $\leqslant$? $\leqslant \Delta^0_1$ (Abdulla et al., 2013)	coNP (Chatterjee et al., 2012)
reachability	Σ^0_1 (Lincoln et al., 1992)	ACK (Urquhart, 1999)

Some Applications of AVASS

- substructural logics
  (Lincoln et al., 1992; Kanovich, 1995; Urquhart, 1999; Lazić and S., 2014)
- energy games (Brázdil et al., 2010; Chatterjee et al., 2012)
- mean-payoff games (Chatterjee et al., 2010)
- one-sided μ-calculus (Abdulla et al., 2013)
- regular simulation games
  (Jančar and Moller, 1995; Lasota, 2009; Abdulla et al., 2013, 2014; Courtois and S., 2014)

Substructural Logics

- Restrict the use of structural rules: e.g.

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}(\mathrm{C}) \qquad \frac{\Gamma \vdash B}{\Gamma, A \vdash B}(\mathrm{W})$$

- track resource usage in logic
- example: relevance logic
  - in $A \to B$, A should be relevant to the proof of B
  - forbids weakening (W) but allows contraction (C)
  - cannot prove e.g. $A \to (B \to A)$ and $(A \& \neg A) \to B$

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(Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathrm{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathrm{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathrm{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathrm{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathrm{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \qquad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathrm{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathrm{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathrm{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathrm{R}_{\oplus})$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}(\mathrm{L}_{\otimes}) \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}(\mathrm{R}_{\otimes})$$

$$\ldots$$

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$$\ldots$$

$(!,\oplus)$-Horn Programs (1/3)

(Kanovich, 1995)

connectives $\{\otimes, \multimap, \oplus, !\}$

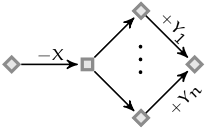
simple products $W, X, Y, Z ::= p_1 \otimes p_2 \otimes \cdots \otimes p_m$
for atomic p_i's

Horn implications $X \multimap Y$

$\oplus$-Horn implications $X \multimap (Y_1 \oplus \cdots \oplus Y_n)$

$(!,\oplus)$-Horn sequents $W, !\Gamma \vdash Z$ where Γ contains
Horn and $\oplus$-Horn implications

$(!, \oplus)$-Horn Programs (2/3)

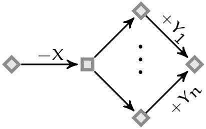
Horn programs		AVASS
$X \multimap Y$	$\Rightarrow$	
$X \multimap (Y_1 \oplus \cdots \oplus Y_n)$	$\Rightarrow$	

$(!, \oplus)$-Horn Programs (2/3)

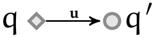
Horn programs | AVASS

$X \multimap Y \quad \Rightarrow$

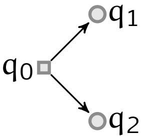
$X \multimap (Y_1 \oplus \cdots \oplus Y_n) \quad \Rightarrow$



$q \otimes \mathbf{u}^- \multimap q' \otimes \mathbf{u}^+ \quad \Leftarrow$



$q_0 \multimap (q_1 \oplus q_2) \quad \Leftarrow$



$(!,\oplus)$-Horn Programs (3/3)

Theorem (Kanovich, 1995)
Provability of $(!,\oplus)$-Horn sequents and AVASS reachability are PSPACE *equivalent.*

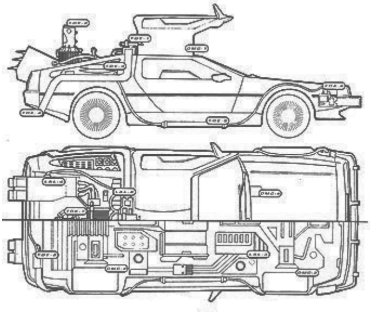
Corollary (Lincoln et al., 1992)
Provability in propositional linear logic is undecidable.

Controller Synthesis

Property

$$F^{-1}F\,p$$

Environment



Controller Synthesis

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$$\Downarrow$$

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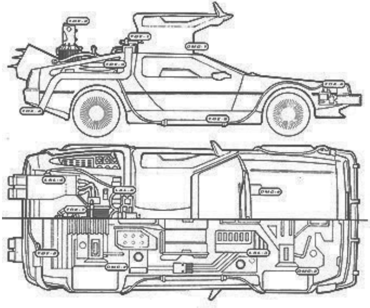
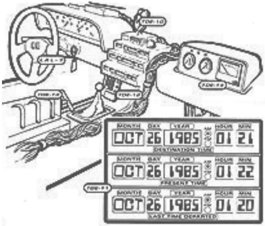


$$\Leftarrow$$

Controller



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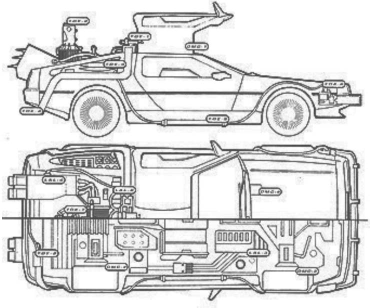
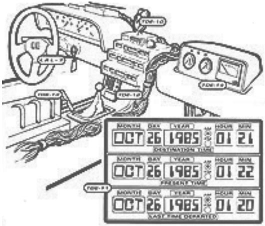


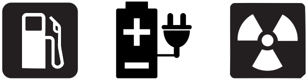
$\Leftarrow$

Controller



Resources

$\Leftarrow$

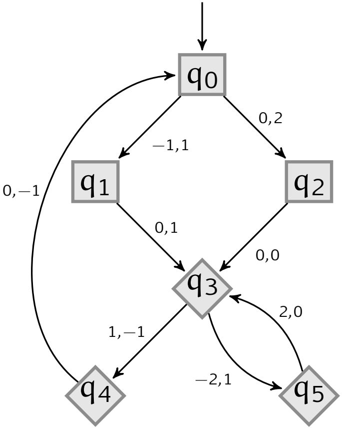


must remain non-negative

Multidimensional Energy Games (1/2)

(Brázdil, Jančar, and Kučera, 2010; Chatterjee, Doyen, Henzinger, and Raskin, 2010)

- defines an infinite arena in $Q \times \mathbb{Z}^d$
- energy semantics: transitions are non-blocking
- non-termination + energy objective: Controller must keep the values non-negative along an infinite play



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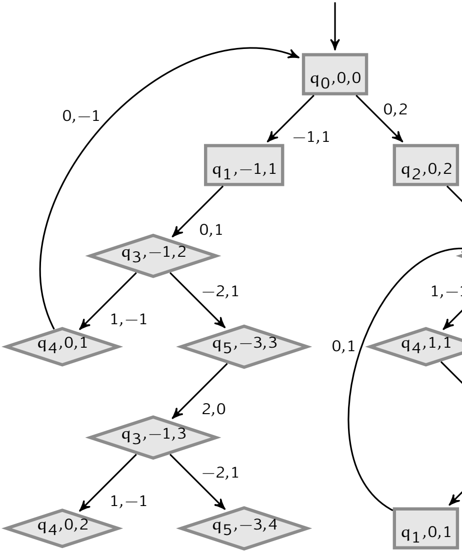
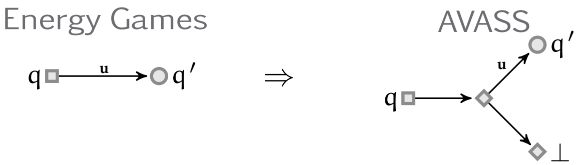


Multidimensional Energy Games (2/2)

(Abdulla, Mayr, Sangnier, and Sproston, 2013)



Theorem (Abdulla et al., 2013)

Non-termination AVASS games and multidimensional energy games are LogSpace*-equivalent.*

Multidimensional Mean-Payoff Games

(Chatterjee, Doyen, Henzinger, and Raskin, 2010)

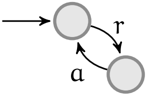
- integer vector game over $Q \times \mathbb{Z}^d$
- payoff: $\liminf_{n\to\infty} \frac{1}{n}\mathbf{v}_n$ if $\mathbf{v}_n$ is the nth vector of the play
- threshold vector $\mathbf{r} \in \mathbb{Q}^d$: a payoff $\geqslant \mathbf{r}$ is sought

Theorem (Chatterjee et al., 2010)

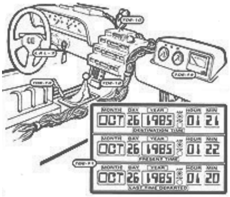
There exists a finite-memory winning strategy for a multidimensional mean-payoff game iff there is a winning strategy in the corresponding multidimensional energy game.

Finite-state specifications

Required behaviours Implementation Safe behaviours



$\preceq$



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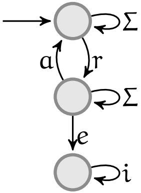

$\models \varphi \in \mathsf{ECTL}^*$

$\models \psi \in \mathsf{ACTL}^*$

Simulation Game

- two labelled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn
  1. Spoiler chooses a successor state in S_1
  2. Duplicator must choose a successor state in S_2 with the same action label
- any blocked player loses; Duplicator wins if the play is infinite

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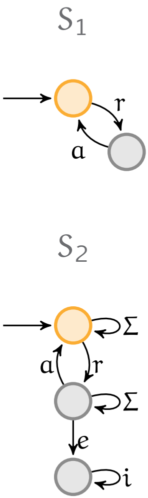

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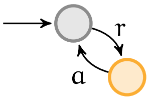
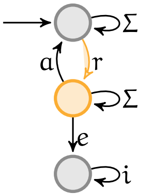


S_2



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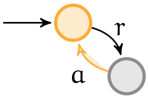
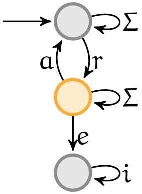


S_2



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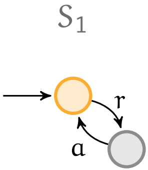

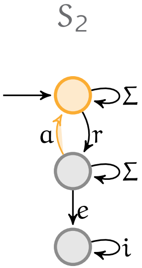

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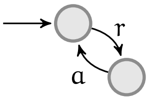

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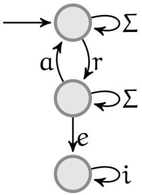


VASS Regular Simulations

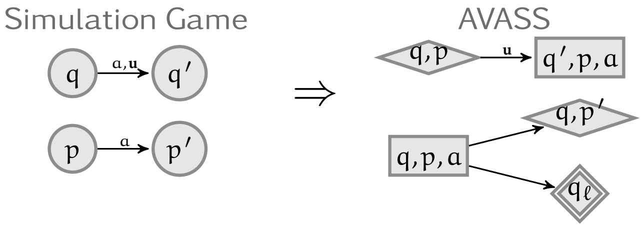
Simulation relations between

- a labelled VASS $\mathcal{V}$ (i.e. an AVASS with $Q_\Box = \emptyset$)
- a finite-state system $\mathcal{F}$

Theorem (Jančar and Moller, 1995)
Both $\mathcal{V} \preceq \mathcal{F}$ and $\mathcal{F} \preceq \mathcal{V}$ are decidable.

Theorem (Lasota, 2009)
Both $\mathcal{V} \preceq \mathcal{F}$ and $\mathcal{F} \preceq \mathcal{V}$ are ExpSpace*-hard, already if $\mathcal{V}$ is a BPP.*

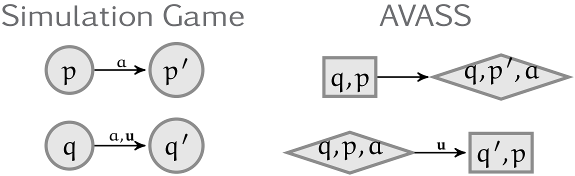
VASS $\preceq$ FS



Theorem (Courtois and S., 2014)

$\mathcal{V} \not\preceq \mathcal{F}$ and coverability AVASS games are LogSpace*-equivalent (already holds for BPP).*

FS $\preceq$ VASS



Theorem (Abdulla et al., 2013; Courtois and S., 2014)

$\mathcal{F} \preceq \mathcal{V}$ and non-termination AVASS games are LogSpace*-equivalent (already holds for BPP).*

Complexity

	initial credit	
objective	fixed	unknown
coverability	2EXP (Courtois and S., 2014)	P (trivial)
non-termination	2EXP $\leqslant$? $\leqslant$ TOWER (Brázdil et al., 2010)	coNP (Chatterjee et al., 2010)
parity	2EXP $\leqslant$? $\leqslant$ Δ^0_1 (Abdulla et al., 2013)	coNP (Chatterjee et al., 2012)
reachability	Σ^0_1 (Lincoln et al., 1992)	ACK (Urquhart, 1999)

Coverability with Fixed Initial Credit

(Courtois and S., 2014)

Proposition (Lower Bounds)

*AVASS Coverability and Non-termination are 2*EXP*-hard, and* EXP*-hard in fixed dimension* $d \geqslant 4$.

Proposition (Upper Bound)

*AVASS Coverability is in 2*EXP*, and in* EXP *in fixed dimension (more precisely pseudo-polynomial).*

Coverability with Fixed Initial Credit

(Courtois and S., 2014)

upper bound Rackoff (1978)'s technique: small witness property

lower bounds Lipton (1976)'s technique: reduction from alternating Minsky machines

Proof Plan for Upper Bound

- if coverable, then there exists a small witness of double exponential height
  - alternating TM can check the existence of a witness in AExpSpace = 2Exp
- induction on dimension: i-witness for $(q, \mathbf{v})$
  - root label $q, \mathbf{v}$
  - enforces coverability: every leaf labelled by q_ℓ
  - allows negative values on coordinates $i < j \leqslant d$
- H_i: bound on $\sup_{q,\mathbf{v}}$ of the heights of minimal i-witnesses for $(q, \mathbf{v})$

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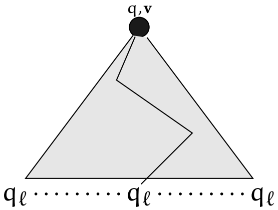
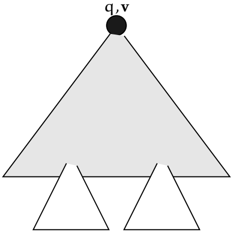
Small Witnesses: Base Case



No state can appear twice along a branch of a minimal 0-witness:

$$H_0 = |Q|$$

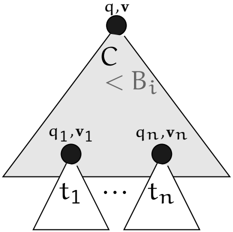
Small Witnesses: Induction Step



an $(i+1)$-witness t

Small Witnesses: Induction Step

$$B_i \stackrel{\text{def}}{=} \|T_u\|_\infty \cdot H_i$$



an $(i+1)$-witness $t = C[t_1, \ldots, t_n]$
$\forall 1 \leqslant j \leqslant n. \exists 1 \leqslant k \leqslant d. \mathbf{v}_j(k) \geqslant B_i$

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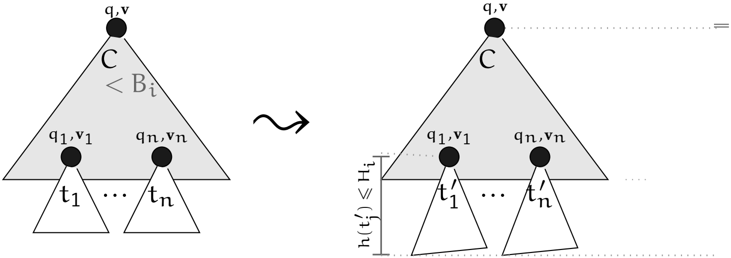

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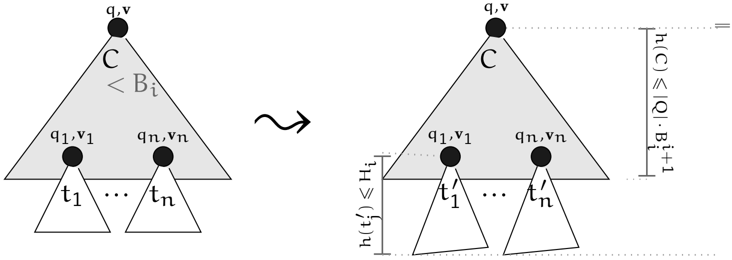

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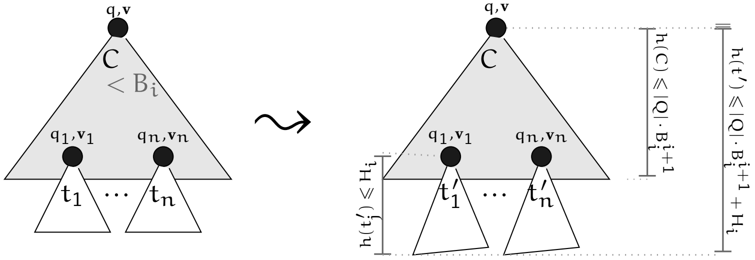

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$t' = C[t'_1,\ldots,t'_n]$

$$H_{i+1} \leqslant |Q| \cdot B_i^{i+1} + H_i$$

Complexity of Non-Termination

Marcin Jurdziński and Ranko Lazić:

Claim

*Non-termination AVASS games with fixed initial credit are in 2*Exp.

This relies on a new bound:

Claim

Non-termination AVASS games with unknown initial credit and fixed dimension d are pseudo-polynomial.

CONCLUDING REMARKS

- alternating VASS / asymmetric VASS games as a sensible model for counter games
- forgotten connections with substructural logics
- upcoming 2EXP-completeness for non-termination AVASS games and FS $\preceq$ VASS
- open gap for parity AVASS games

Abdulla, P.A., Mayr, R., Sangnier, A., and Sproston, J., 2013. Solving parity games on integer vectors. In D'Argenio, P.R. and Melgratti, H., editors, *Concur 2013*, volume 8052 of *LNCS*, pages 106–120. Springer. doi:10.1007/978-3-642-40184-8_9.

Abdulla, P.A., Atig, M.F., Hofman, P., Mayr, R., Kumar, K.N., and Totzke, P., 2014. Infinite-state energy games. In *CSL-LICS 2014*, pages 7:1–7:10. ACM. doi:10.1145/2603088.2603100.

Brázdil, T., Jančar, P., and Kučera, A., 2010. Reachability games on extended vector addition systems with states. In Abramsky, S., Gavoille, C., Kirchner, C., Meyer auf der Heide, F., and Spirakis, P.G., editors, *ICALP 2010*, volume 6199 of *LNCS*, pages 478–489. Springer. doi:10.1007/978-3-642-14162-1_40.

Chatterjee, K., Doyen, L., Henzinger, T.A., and Raskin, J.F., 2010. Generalized mean-payoff and energy games. In Lodaya, K. and Mahajan, M., editors, *FSTTCS 2010*, volume 8 of *LIPIcs*, pages 505–516. LZI. doi:10.4230/LIPIcs.FSTTCS.2010.505.

Chatterjee, K., Randour, M., and Raskin, J.F., 2012. Strategy synthesis for multi-dimensional quantitative objectives. In Koutny, M. and Ulidowski, I., editors, *Concur 2012*, volume 7454 of *LNCS*, pages 115–131. Springer. doi:10.1007/978-3-642-32940-1_10.

Courtois, J.B. and Schmitz, S., 2014. Alternating vector addition systems with states. In Csuhaj-Varjú, E., Dietzfelbinger, M., and Ésik, Z., editors, *MFCS 2014*, volume 8634 of *LNCS*, pages 220–231. Springer. doi:10.1007/978-3-662-44522-8_19.

Figueira, D., Figueira, S., Schmitz, S., and Schnoebelen, Ph., 2011. Ackermannian and primitive-recursive bounds with Dickson's Lemma. In *LICS 2011*, pages 269–278. IEEE Computer Society. doi:10.1109/LICS.2011.39.

Jančar, P. and Moller, F., 1995. Checking regular properties of Petri nets. In Lee, I. and Smolka, S.A., editors, *Concur '95*, volume 962 of *LNCS*, pages 348–362. Springer. doi:10.1007/3-540-60218-6_26.

Kanovich, M.I., 1995. Petri nets, Horn programs, linear logic and vector games. *Ann. Pure App. Logic*, 75(1–2): 107–135. doi:10.1016/0168-0072(94)00060-G.

Lasota, S., 2009. ExpSpace lower bounds for the simulation preorder between a communication-free Petri net and a finite-state system. *Information Processing Letters*, 109(15):850–855. doi:10.1016/j.ipl.2009.04.003.

Lazić, R. and Schmitz, S., 2014. Non-elementary complexities for branching VASS, MELL, and extensions. In *CSL-LICS 2014*, pages 61:1–61:10. ACM. doi:10.1145/2603088.2603129.

Lincoln, P., Mitchell, J., Scedrov, A., and Shankar, N., 1992. Decision problems for propositional linear logic. *Ann. Pure App. Logic*, 56(1–3):239–311. doi:10.1016/0168-0072(92)90075-B.

Lipton, R., 1976. The reachability problem requires exponential space. Technical Report 62, Yale University.

Rackoff, C., 1978. The covering and boundedness problems for vector addition systems. *Theor. Comput. Sci.*, 6(2): 223–231. doi:10.1016/0304-3975(78)90036-1.

Raskin, J.F., Samuelides, M., and Van Begin, L., 2005. Games for counting abstractions. In *AVoCS 2004*, volume 128(6) of *ENTCS*, pages 69–85. Elsevier. doi:10.1016/j.entcs.2005.04.005.

Urquhart, A., 1999. The complexity of decision procedures in relevance logic II. *J. Symb. Log.*, 64(4):1774–1802. doi:10.2307/2586811.