The Complexity of Vector Addition Games

work in progress!

J.-B. Courtois S. Schmitz

LSV, ENS Cachan, France

68nqrt Seminar, November 7th 2013

Outline

vector addition games and alternating VASS

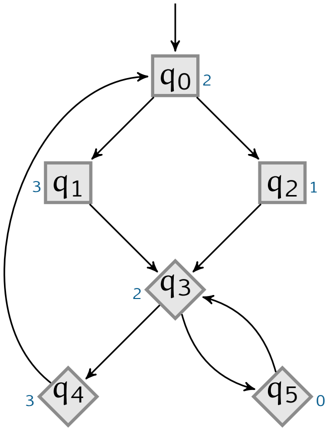
applications

- substructural logics
- energy games
- regular simulations

upper bounds

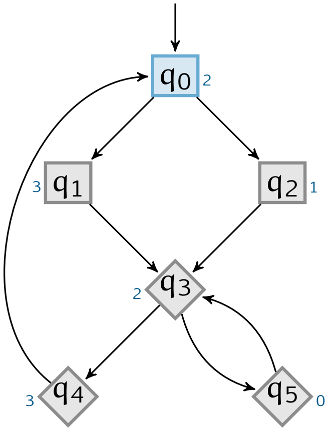
Parity Games

- two Players $\Diamond$ and $\Box$
- partitioned state space $Q_\Diamond \uplus Q_\Box$
- priority in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest priority seen infinitely often is even



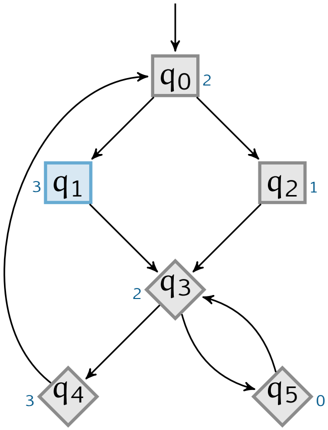
Parity Games

- two Players $\Diamond$ and $\Box$
- partitioned state space $Q_\Diamond \uplus Q_\Box$
- priority in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest priority seen infinitely often is even



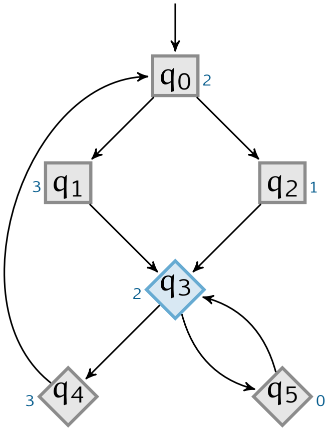
Parity Games

- two Players $\Diamond$ and $\Box$
- partitioned state space $Q_\Diamond \uplus Q_\Box$
- priority in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest priority seen infinitely often is even



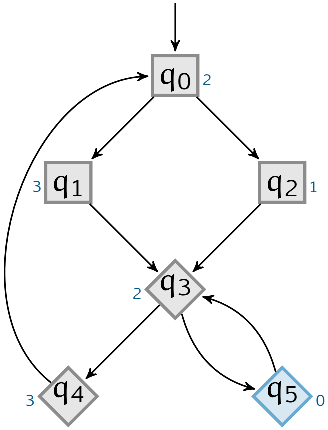
Parity Games

- two Players $\Diamond$ and $\Box$
- partitioned state space $Q_\Diamond \uplus Q_\Box$
- priority in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest priority seen infinitely often is even



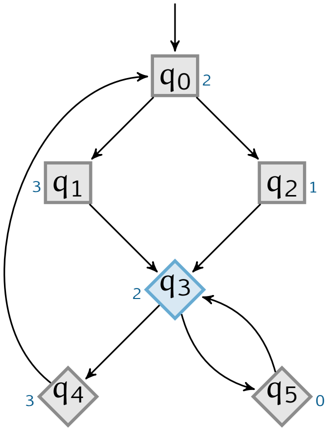
Parity Games

- two Players $\Diamond$ and $\Box$
- partitioned state space $Q_{\Diamond} \uplus Q_{\Box}$
- priority in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest priority seen infinitely often is even



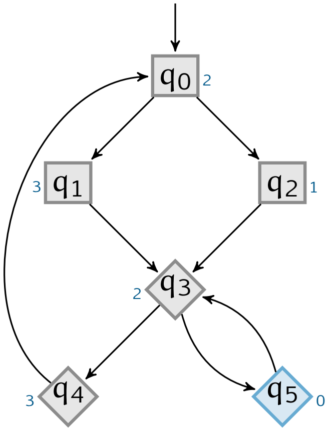
Parity Games

- two Players $\Diamond$ and $\Box$
- partitioned state space $Q_\Diamond \uplus Q_\Box$
- priority in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest priority seen infinitely often is even



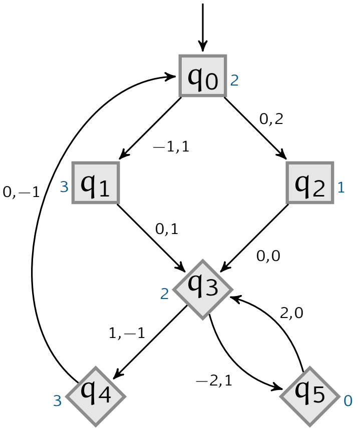
Parity Games

- two Players $\Diamond$ and $\Box$
- partitioned state space $Q_\Diamond \uplus Q_\Box$
- priority in $\{1,\ldots,k\}$ on each state
- parity objective: Player $\Diamond$ wins iff the smallest priority seen infinitely often is even



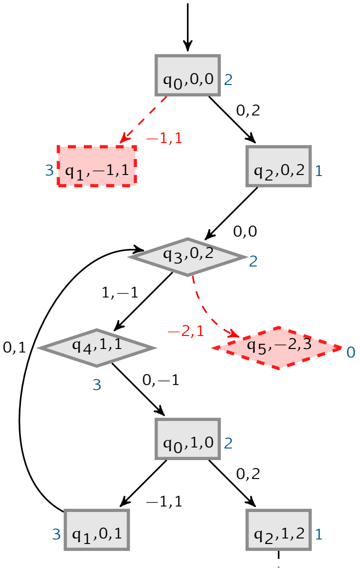
Vector Addition Games

- transitions labeled with vectors in $\mathbb{Z}^d$
- defines an infinite arena in $Q \times \mathbb{N}^d$
- VASS semantics: a transition is blocked if it makes a value negative



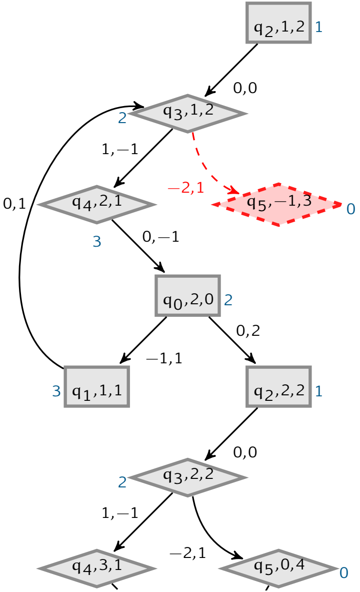
Vector Addition Games

- transitions labeled with vectors in $\mathbb{Z}^d$
- defines an infinite arena in $Q \times \mathbb{N}^d$
- VASS semantics: a transition is blocked if it makes a value negative



Vector Addition Games

- transitions labeled with vectors in $\mathbb{Z}^d$
- defines an infinite arena in $Q \times \mathbb{N}^d$
- VASS semantics: a transition is blocked if it makes a value negative



Asymmetric VASS Games

aka. vector games (Kanovich, 1995), B-games (Raskin et al., 2005), single-sided games (Abdulla et al., 2013)

Asymmetric VASS (AVASS) game:

- $Q = Q_\Diamond \uplus Q_\Box$, resp. Control and Environment
- $T_\Diamond \subseteq Q_\Diamond \times \mathbb{Z}^d \times Q$:

$$q_\Diamond \xrightarrow{\delta} q'$$

- $T_\Box \subseteq Q_\Box \times \{\mathbf{0}\} \times Q$:

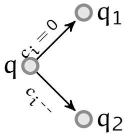
$$\begin{array}{ccc} & & q_1 \\ & \nearrow & \\ q_\Box & & \\ & \searrow & \\ & & q_2 \end{array}$$

The Importance of Asymmetry

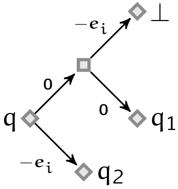
(Raskin et al., 2005)

- VASS game: $T \subseteq Q \times \mathbb{Z}^d \times Q$
- coverability objective: fix q_f, target $\{q_f\} \times \mathbb{N}^d$

Minsky machine



Symmetric VASS Game



Player □ can simulate zero-tests!

Alternating VASS

aka. "and-branching" (Lincoln et al., 1992; Urquhart, 1999)

Q finite set of states

q_0 initial state in Q

T_1 finite set of unary transitions $\subseteq Q \times \mathbb{Z}^d \times Q$:

$$q \xrightarrow{\delta} q'$$

T_2 set of fork transitions $\subseteq Q^3$:

$$\begin{tikzcd}[row sep=small] & q_1 \\ q \arrow[ur] \arrow[dr] & \\ & q_2 \end{tikzcd}$$

Tree Semantics

run in $T(Q \times \mathbb{N}^d)$:

Initial

$q_0, \mathbf{0}$

T_1

$$\begin{tikzcd} q, \boldsymbol{v} \arrow[d, "\delta"] \\ q', \boldsymbol{v} + \delta \end{tikzcd}$$

blocking

transition:

$\boldsymbol{v} + \delta \geqslant \mathbf{0}$

T_2

$$\begin{tikzcd} & q, \boldsymbol{v} \arrow[dl] \arrow[dr] & \\ q_1, \boldsymbol{v} & & q_2, \boldsymbol{v} \end{tikzcd}$$

different possible acceptance conditions
(on branches)

Some Applications of AVASS

- propositional linear logic (Lincoln et al., 1992; Kanovich, 1995)
- relevance logic (Urquhart, 1999)
- multidimensional energy games (Brázdil et al., 2010; Chatterjee et al., 2012)
- multidimensional mean-payoff games (Chatterjee et al., 2010)
- one-sided μ-calculus (Abdulla et al., 2013)
- regular simulation games (Jančar and Moller, 1995; Lasota, 2009; Abdulla et al., 2013)

Substructural Logics

- Restrict the use of structural rules: e.g.

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}(C) \qquad \frac{\Gamma \vdash B}{\Gamma, A \vdash B}(W)$$

- track resource usage in logic
- example: relevance logic
  - in $A \to B$, A should be relevant to the proof of B
  - forbids weakening (W) but allows contraction (C)
  - cannot prove e.g. $A \to (B \to A)$ and $(A \& \neg A) \to B$

Substructural Logics

- Restrict the use of structural rules: e.g.

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}(C) \qquad \frac{\Gamma \vdash B}{\Gamma, A \vdash B}(W)$$

- track resource usage in logic
- example: relevance logic
  - in $A \to B$, A should be relevant to the proof of B
  - forbids weakening (W) but allows contraction (C)
  - cannot prove e.g. $A \to (B \to A)$ and $(A \& \neg A) \to B$

(Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathsf{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathsf{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathsf{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathsf{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathsf{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \qquad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathsf{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathsf{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathsf{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathsf{R}_{\oplus})$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}(\mathsf{L}_{\otimes}) \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}(\mathsf{R}_{\otimes})$$

$$\ldots$$

(Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathrm{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathrm{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathrm{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathrm{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathrm{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \quad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathrm{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathrm{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathrm{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathrm{R}_{\oplus})$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}(\mathrm{L}_{\otimes}) \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}(\mathrm{R}_{\otimes})$$

...

(Intuitionistic) Linear Logic

$$\frac{}{A \vdash A}(\mathrm{I}) \qquad \frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}(\mathrm{C!}) \qquad \frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}(\mathrm{L!})$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}(\mathrm{L}_{\multimap}) \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}(\mathrm{R}_{\multimap})$$

$$\frac{\Gamma, A \vdash C}{\Gamma, A \& B \vdash C} \quad \frac{\Gamma, B \vdash C}{\Gamma, A \& B \vdash C}(\mathrm{L}_{\&}) \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}(\mathrm{R}_{\&})$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \oplus B \vdash C}(\mathrm{L}_{\oplus}) \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \oplus B} \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \oplus B}(\mathrm{R}_{\oplus})$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}(\mathrm{L}_{\otimes}) \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}(\mathrm{R}_{\otimes})$$

$$\ldots$$

$(!, \oplus)$-Horn Programs (1/3)

connectives $\{\otimes, \multimap, \oplus, !\}$

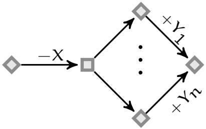
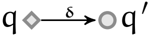
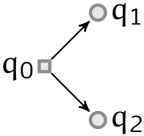
simple products $W, X, Y, Z ::= p_1 \otimes p_2 \otimes \cdots \otimes p_m$
for atomic p_i's

Horn implications $X \multimap Y$

$\oplus$-Horn implications $X \multimap (Y_1 \oplus \cdots \oplus Y_n)$

$(!, \oplus)$-Horn sequents $W, !\Gamma \vdash Z$ where Γ contains
Horn and $\oplus$-Horn implications

$(!, \oplus)$-Horn Programs (2/3)

Horn programs		AVASS
$X \multimap Y$	$\Rightarrow$	
$X \multimap (Y_1 \oplus \cdots \oplus Y_n)$	$\Rightarrow$	
$q \otimes \delta^- \multimap q' \otimes \delta^+$	$\Leftarrow$	
$q_0 \multimap (q_1 \oplus q_2)$	$\Leftarrow$	

$(!, \oplus)$-Horn Programs (3/3)

AVASS Reachability

input AVASS $\mathcal{A}$, configuration $q, \mathbf{v} \in Q \times \mathbb{N}^d$

question can Controller win for the reachability objective $\{(q, \mathbf{v})\}$?

Theorem (Lincoln et al., 1992; Kanovich, 1995; Raskin et al., 2005)

AVASS Reachability is undecidable.

Corollary (Lincoln et al., 1992)

Provability in propositional linear logic is undecidable.

Corollary (Kanovich, 1995)

Provability of $(!, \oplus)$-Horn sequents is undecidable.

$(!,\oplus)$-Horn Programs (3/3)

AVASS Reachability

input AVASS $\mathcal{A}$, configuration $q,\boldsymbol{v} \in Q \times \mathbb{N}^d$

question can Controller win for the reachability objective $\{(q,\boldsymbol{v})\}$?

Theorem (Lincoln et al., 1992; Kanovich, 1995; Raskin et al., 2005)

AVASS Reachability is undecidable.

Corollary (Lincoln et al., 1992)

Provability in propositional linear logic is undecidable.

Corollary (Kanovich, 1995)

Provability of $(!,\oplus)$-Horn sequents is undecidable.

Relevance Logic $R_{\&,\multimap}$ (1/2)

connectives $\{\oplus, \otimes, \&, \multimap\}$

rules with contraction (C)

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}\text{(C)}$$

but without weakening (W)

Relevance Logic $R_{\&,\multimap}$ (2/2)

add increasing transitions to account for (C):

$\forall q \in Q_\Diamond, \forall i \leqslant d. q \xrightarrow{e_i} q$

AVASS "Bottom-up" Coverability

input increasing AVASS $\mathcal{A}$ and configuration $q, \mathbf{v} \in Q_\Diamond \times \mathbb{N}^d$

question can Controller win for the reachability objective $\{(q, \mathbf{v})\}$?

Theorem (Urquhart, 1999)

Bottom-up Coverability is Ackermann*-complete.*

Corollary (Urquhart, 1999)

Provability in $R_{\&,\multimap}$ *is* Ackermann*-complete.*

Relevance Logic $R_{\&,\multimap}$ (2/2)

add increasing transitions to account for (C):

$\forall q \in Q_\Diamond, \forall i \leqslant d. q \xrightarrow{e_i} q$

AVASS "Bottom-up" Coverability

input increasing AVASS $\mathcal{A}$ and configuration $q, \boldsymbol{v} \in Q_\Diamond \times \mathbb{N}^d$

question can Controller win for the reachability objective $\{(q, \boldsymbol{v})\}$?

Theorem (Urquhart, 1999)

Bottom-up Coverability is Ackermann*-complete.*

Corollary (Urquhart, 1999)

Provability in $R_{\&,\multimap}$ *is* Ackermann*-complete.*

Relevance Logic $R_{\&,\multimap}$ (2/2)

add increasing transitions to account for (C):

$\forall q \in Q_\diamond, \forall i \leqslant d. q \xrightarrow{e_i} q$

AVASS "Bottom-up" Coverability

input increasing AVASS $\mathcal{A}$ and configuration $q, v \in Q_\diamond \times \mathbb{N}^d$

question can Controller win for the reachability objective $\{(q, v)\}$?

Theorem (Urquhart, 1999)

Bottom-up Coverability is Ackermann*-complete.*

Corollary (Urquhart, 1999)

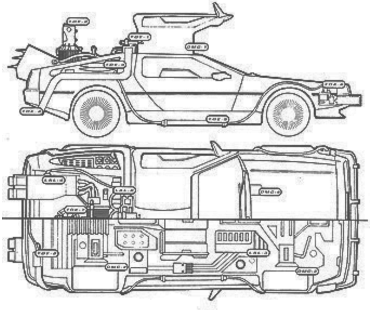
Provability in $R_{\&,\multimap}$ *is* Ackermann*-complete.*

Controller Synthesis

Property

$$F^{-1}F\,p$$

Environment



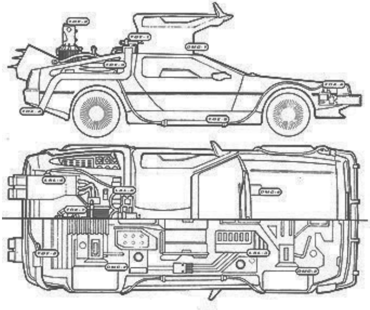
Controller Synthesis

Property

$$F^{-1}F\,p$$

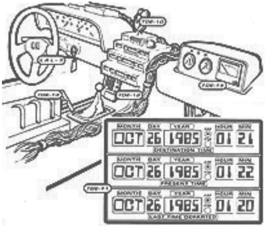
$$\Downarrow$$

Environment



$\Leftarrow$

Controller



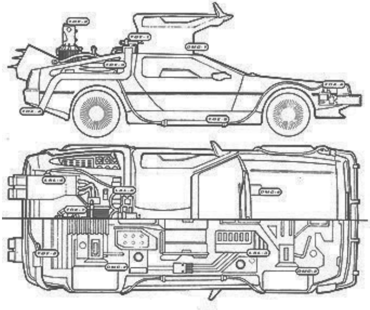
Controller Synthesis

Property

$$F^{-1}F\,p$$

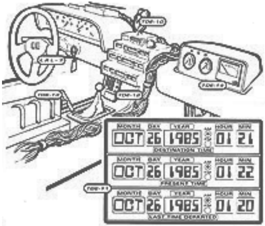
$$\Downarrow$$

Environment



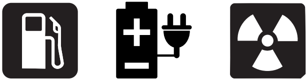
$\Leftarrow$

Controller



Resources

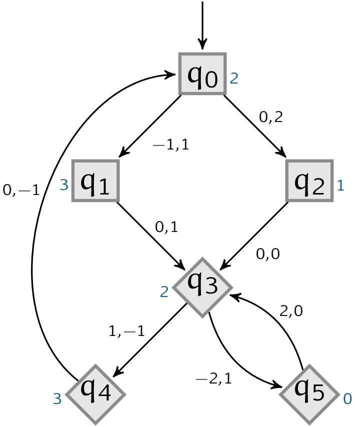
$\Leftarrow$



must remain non-negative

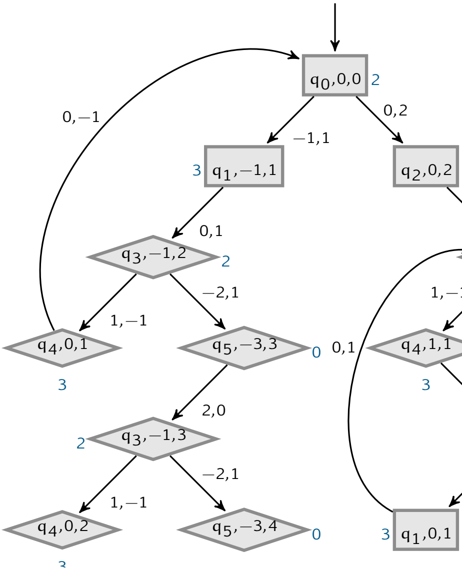
Multidimensional Energy Games (1/3)

- defines an infinite arena in $Q \times \mathbb{Z}^d$
- energy semantics: transitions are non-blocking
- energy objective: Controller must keep the values non-negative



Multidimensional Energy Games (1/3)

- defines an infinite arena in $Q \times \mathbb{Z}^d$
- energy semantics: transitions are non-blocking
- energy objective: Controller must keep the values non-negative



Multidimensional Energy Games (2/3)

Parity

input AVASS $\mathcal{A}$ and priority assignment $c: Q \to \mathbb{N}$

question can Controller win for the parity objective?

Theorem (Abdulla et al., 2013)
AVASS Parity is LogSpace-equivalent to multidimensional energy parity games.

Energy Games

AVASS

Multidimensional Energy Games (2/3)

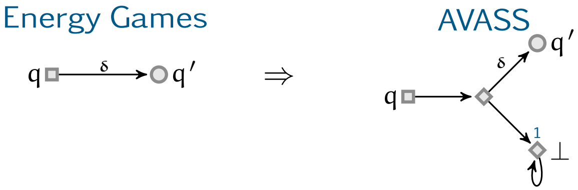
Parity

input AVASS $\mathcal{A}$ and priority assignment $c\colon Q \to \mathbb{N}$

question can Controller win for the parity objective?

Theorem (Abdulla et al., 2013)

AVASS Parity is LogSpace*-equivalent to multidimensional energy parity games.*



Multidimensional Energy Games (3/3)

Unknown Initial Credit

input AVASS $\mathcal{A}$ and priority assignment $c{:}\,Q \to \mathbb{N}$

question $\exists \boldsymbol{v} \in \mathbb{N}^d$ s.t. Controller wins for the parity objective when starting from $q_0, \boldsymbol{v}$?

Theorem (Chatterjee et al., 2012)

AVASS Parity with unknown initial credit is coNP-complete.

Multidimensional Energy Games (3/3)

Unknown Initial Credit

input AVASS $\mathcal{A}$ and priority assignment $c: Q \to \mathbb{N}$

question $\exists \boldsymbol{v} \in \mathbb{N}^d$ s.t. Controller wins for the parity objective when starting from $q_0, \boldsymbol{v}$?

Theorem (Chatterjee et al., 2012)
AVASS $Parity$ with unknown initial credit is coNP-complete.

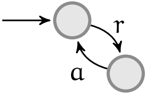
Multidimensional Mean-Payoff Games

- integer vector game over $Q \times \mathbb{Z}^d$
- payoff: $\liminf_{n\to\infty} \frac{1}{n}\boldsymbol{v}_n$ if $\boldsymbol{v}_n$ is the nth vector of the play
- threshold vector $\boldsymbol{r} \in \mathbb{Q}^d$: a payoff $\geqslant \boldsymbol{r}$ is seeked

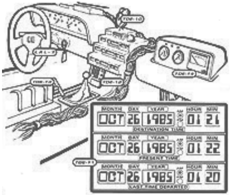
Theorem (Chatterjee et al., 2010)
Finite-memory strategies for multidimensional mean-payoff games with unknown initial credit are coNP-complete.

Finite-state specifications

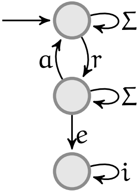
Required behaviours | Implementation | Safe behaviours



$\preceq$



$\preceq$



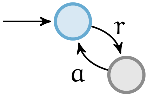
$\models \varphi \in \mathsf{ECTL}^*$

$\models \psi \in \mathsf{ACTL}^*$

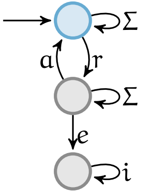
Simulation Game

- two labeled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn
  1. Spoiler chooses a successor state in S_1
  2. Duplicator must choose a successor state in S_2 with the same action label
- any blocked player looses; Duplicator wins if the play is infinite

S_1



S_2



Simulation Game

- two labeled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn

1. Spoiler chooses a successor state in S_1
2. Duplicator must choose a successor state in S_2 with the same action label

- any blocked player looses; Duplicator wins if the play is infinite

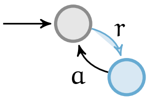
Simulation Game

- two labeled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn

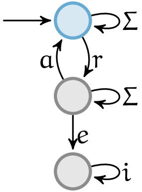
1. Spoiler chooses a successor state in S_1
2. Duplicator must choose a successor state in S_2 with the same action label

- any blocked player looses; Duplicator wins if the play is infinite

S_1



S_2



Simulation Game

- two labeled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn

1. Spoiler chooses a successor state in S_1
2. Duplicator must choose a successor state in S_2 with the same action label

- any blocked player looses; Duplicator wins if the play is infinite

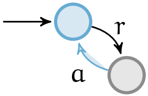
Simulation Game

- two labeled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn

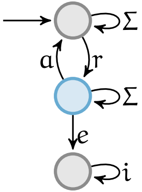
1. Spoiler chooses a successor state in S_1
2. Duplicator must choose a successor state in S_2 with the same action label

- any blocked player looses; Duplicator wins if the play is infinite

S_1



S_2

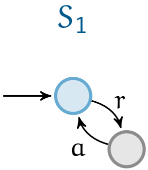


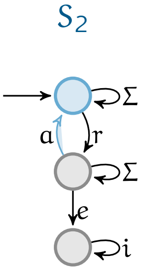
Simulation Game

- two labeled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn

1. Spoiler chooses a successor state in S_1
2. Duplicator must choose a successor state in S_2 with the same action label

- any blocked player looses; Duplicator wins if the play is infinite





Simulation Game

- two labeled transition systems S_1 and S_2
- two players Spoiler and Duplicator
- at each turn

1. Spoiler chooses a successor state in S_1
2. Duplicator must choose a successor state in S_2 with the same action label

- any blocked player looses; Duplicator wins if the play is infinite

VASS Regular Simulations

Simulation relations between

- a labeled VASS $\mathcal{V}$ (i.e. an AVASS with $Q_\Box = \emptyset$)
- a finite-state system $\mathcal{F}$

Theorem (Jančar and Moller, 1995)
$\mathcal{V} \preceq \mathcal{F}$ and $\mathcal{F} \preceq \mathcal{V}$ are decidable.

Theorem (Lasota, 2009)
$\mathcal{V} \preceq \mathcal{F}$ and $\mathcal{F} \preceq \mathcal{V}$ are ExpSpace*-hard, already if $\mathcal{V}$ is a BPP.*

VASS $\preceq$ FS

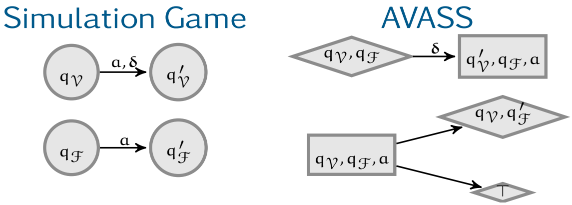
Coverability

input AVASS $\mathcal{A}$ and state q

question can Controller win for the reachability objective $\{q\} \times \mathbb{N}^d$?

Proposition

$\mathcal{V} \not\preceq \mathcal{F}$ and AVASS Coverability *are* LogSpace*-equivalent (already holds for BPP).*



FS $\preceq$ VASS

Non-termination

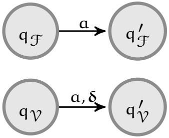
input AVASS $\mathcal{A}$

question can Controller force an infinite play?

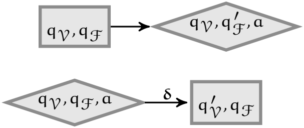
Proposition

$\mathcal{F} \preceq \mathcal{V}$ and AVASS Non-termination are LogSpace*-equivalent (already holds for BPP).*

Simulation Game



AVASS



Key Problem: Parity

Theorem (Brázdil et al., 2010)

AVASS Non-termination is in Tower *and* ExpSpace*-hard.*

Conjecture (in progress)

*AVASS Non-termination, Coverability, and Parity are 2*ExpTime*-complete.*

Currently: Coverability is 2ExpTime-complete, Non-termination is 2ExpTime-hard:

upper bound Rackoff (1978)'s technique, plus insights from (Brázdil et al., 2010)

lower bounds Lipton (1976)'s technique

Key Problem: Parity

Theorem (Brázdil et al., 2010)

AVASS Non-termination is in Tower *and* ExpSpace*-hard.*

Conjecture (in progress)

*AVASS Non-termination, Coverability, and Parity are 2*ExpTime*-complete.*

Currently: Coverability is 2ExpTime-complete, Non-termination is 2ExpTime-hard:

upper bound Rackoff (1978)'s technique, plus insights from (Brázdil et al., 2010)

lower bounds Lipton (1976)'s technique

Key Problem: Parity

Theorem (Brázdil et al., 2010)

AVASS Non-termination is in Tower *and* ExpSpace*-hard.*

Conjecture (in progress)

*AVASS Non-termination, Coverability, and Parity are 2*ExpTime*-complete.*

Currently: Coverability is 2ExpTime-complete, Non-termination is 2ExpTime-hard:

upper bound Rackoff (1978)'s technique, plus insights from (Brázdil et al., 2010)

lower bounds Lipton (1976)'s technique

PROOF PLAN FOR COVERABILITY

- existence of "small" witnesses (double exponential depth)
  - alternating TM can check the existence of a witness in 2EXPTIME
- induction on dimension
  - i-witness: enforces objective, but allow negative values on coordinates $i < j \leqslant d$
- (... proof on whiteboard)

Proof Plan for Coverability

- existence of “small” witnesses (double exponential depth)
  - alternating TM can check the existence of a witness in 2ExpTime
- induction on dimension
  - i-witness: enforces objective, but allow negative values on coordinates $i < j \leqslant d$
- (... proof on whiteboard)

Proof Plan for Coverability

- existence of "small" witnesses (double exponential depth)
  - alternating TM can check the existence of a witness in 2ExpTime
- induction on dimension
  - i-witness: enforces objective, but allow negative values on coordinates $i < j \leqslant d$
- (... proof on whiteboard)

Concluding Remarks

- alternating VASS / asymmetric VASS games as a sensible model for counter games
- forgotten connections with substructural logics
- importance of Rackoff (1978)'s techniques

References

Abdulla, P.A., Mayr, R., Sangnier, A., and Sproston, J., 2013. Solving parity games on integer vectors. In D'Argenio, P.R. and Melgratti, H., editors, *Concur 2013*, volume 8052 of *Lect. Notes in Comput. Sci.*, pages 106–120. Springer. doi:10.1007/978-3-642-40184-8_9.

Brázdil, T., Jančar, P., and Kučera, A., 2010. Reachability games on extended vector addition systems with states. In Abramsky, S., Gavoille, C., Kirchner, C., Meyer auf der Heide, F., and Spirakis, P.G., editors, *ICALP 2010*, volume 6199 of *Lect. Notes in Comput. Sci.*, pages 478–489. Springer. doi:10.1007/978-3-642-14162-1_40.

Chatterjee, K., Doyen, L., Henzinger, T.A., and Raskin, J.F., 2010. Generalized mean-payoff and energy games. In Lodaya, K. and Mahajan, M., editors, *FSTTCS 2010*, volume 8 of *Leibniz Int. Proc. Inf.*, pages 505–516. LZI. doi:10.4230/LIPIcs.FSTTCS.2010.505.

Chatterjee, K., Randour, M., and Raskin, J.F., 2012. Strategy synthesis for multi-dimensional quantitative objectives. In Koutny, M. and Ulidowski, I., editors, *Concur 2012*, volume 7454 of *Lect. Notes in Comput. Sci.*, pages 115–131. Springer. doi:10.1007/978-3-642-32940-1_10.

Habermehl, P., 1997. On the complexity of the linear-time μ-calculus for Petri nets. In Azéma, P. and Balbo, G., editors, *Petri Nets '97*, volume 1248 of *Lect. Notes in Comput. Sci.*, pages 102–116. Springer. doi:10.1007/3-540-63139-9_32.

Jančar, P. and Moller, F., 1995. Checking regular properties of Petri nets. In Lee, I. and Smolka, S.A., editors, *Concur '95*, volume 962 of *Lect. Notes in Comput. Sci.*, pages 348–362. Springer. doi:10.1007/3-540-60218-6_26.

Kanovich, M.I., 1995. Petri nets, Horn programs, linear logic and vector games. *Ann. Pure App. Logic*, 75(1–2): 107–135. doi:10.1016/0168-0072(94)00060-G.

Lasota, S., 2009. ExpSpace lower bounds for the simulation preorder between a communication-free Petri net and a finite-state system. *Information Processing Letters*, 109(15):850–855. doi:10.1016/j.ipl.2009.04.003.

Lincoln, P., Mitchell, J., Scedrov, A., and Shankar, N., 1992. Decision problems for propositional linear logic. *Ann. Pure App. Logic*, 56(1–3):239–311. doi:10.1016/0168-0072(92)90075-B.

Lipton, R., 1976. The reachability problem requires exponential space. Technical Report 62, Yale University.

Rackoff, C., 1978. The covering and boundedness problems for vector addition systems. *Theor. Comput. Sci.*, 6(2): 223–231. doi:10.1016/0304-3975(78)90036-1.

Raskin, J.F., Samuelides, M., and Begin, L.V., 2005. Games for counting abstractions. In *AVoCS 2004*, volume 128(6) of *Elec. Notes in Theor. Comput. Sci.*, pages 69–85. Elsevier. doi:10.1016/j.entcs.2005.04.005.

Urquhart, A., 1999. The complexity of decision procedures in relevance logic II. *J. Symb. Log.*, 64(4):1774–1802. doi:10.2307/2586811.