# Complexité avancée - TD 3

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**Exercise 1 Space hierarchy theorem**

Consider two space-constructible functions $f$ and $g$ such that $f(n) = o(g(n))$. Prove that $\mathsf{DSPACE}(f) \subsetneq \mathsf{DSPACE}(g)$.

*Hint: You may consider a language $L = \{(M, w') \mid$ the simulation of $M$ on $(M, w')$ rejects $\}$ with an appropriate restriction on the simulation of $M$.*

**Exercise 2 Polylogarithmic space**

1. Let $\mathsf{polyL} = \cup_{k \in \mathbb{N}} \mathsf{SPACE}(\log^k)$. Show that $\mathsf{polyL}$ does not have a complete problem for logarithmic space reduction.[1]

2. We recall that $\mathsf{PSPACE} = \cup_{k \in \mathbb{N}} \mathsf{SPACE}(n^k)$. Does $\mathsf{PSPACE}$ have a complete problem for logarithmic space reduction ? Why doesn't the proof of the previous question apply to $\mathsf{PSPACE}$?

**Exercise 3 Padding argument**

1. Show that if $\mathsf{DSPACE}(n^c) \subseteq \mathsf{NP}$ for some $c > 0$, then $\mathsf{PSPACE} \subseteq \mathsf{NP}$.

   *Hint: for $\mathsf{L} \in \mathsf{DSPACE}(n^k)$ one may consider the language $\tilde{L} = \{(x, 1^{|x|^{k/c}}) \mid x \in L\}$.*

2. Deduce that $\mathsf{DSPACE}(n^c) \neq \mathsf{NP}$.

[1] From this, we can deduce that $\mathsf{polyL} \neq \mathsf{P}$.