Complexité avancée - Homework 5

Benjamin Bordais

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Due at 11.59 p.m., November 2, 2020

Some P-complete Problems

1. Show that the following problems are P-hard.

   - Recall that for a finite set X, a subset $S \subseteq X$, and a binary operator $* : X \times X \to X$ defined on X, we inductively define $S_{0,*} = S$ and $S_{i+1,*} = S_{i,*} \cup \{x * y \mid x, y \in S_{i,*}\}$. Then, the closure of S with regard to $*$ is the set $S_* = \cup_{i \in \mathbb{N}} S_{i,*}$.

     BinOpGen:

     - INPUT: A finite set X, a binary operator $* : X \times X \to X$ defined on X, a subset $S \subset X$ and $x \in X$;
     - OUTPUT: $x \in S_*$?

     Hint: reduce from MonotoneCircuitValue *with all nodes having at most two predecessors.*

   - Recall that a context-free grammar is a grammar $G = (V, A, S, \mathcal{P})$ where V is the set of non-terminal symbols, A is the alphabet of terminal symbols, $S \in V$ is the axiom and $\mathcal{P} \subseteq V \times (V \cup A)^*$ is the finite set of production rules (the "context-free" part can be seen in the fact that the left-hand member of a rule in $\mathcal{P}$ has length one). The language $\mathcal{L}(G)$ of G is the set of words $w \in A^*$ that can be derived from S by applying the production rules.

     CFG-Derivability:

     - INPUT: G a context-free grammar on an alphabet A, and w a word;
     - QUESTION: $w \in \mathcal{L}(G)$?

     Hint: reduce from the previous problem.

2. In fact these problems are P-complete. Show that BinOpGen is in P. Do you know a polynomial-time algorithm for CFG-Derivability ?