

# Complexité avancée - Homework 3

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Due at 8.30 a.m., October 14, 2020

**PSPACE and games** The Geography game is played as follows:

- The game starts with a given name of a city, for instance *Cachan*;
- the first player gives the name of a city whose first letter coincides with the last letter of the previous city, for instance *Nice*;
- the second player gives then another city name, also starting with the last letter of the previous city, for instance *Evy*;
- the first player plays again, and so on – with the restriction that no player is allowed to give the name of a city already used in the game;
- the loser is the first player who does not find a new city name to continue.

This game can be described using a directed graph whose vertices represent cities and where an edge  $(X, Y)$  means that the last letter of the city  $X$  is the same as the first letter of the city  $Y$ . This graph has also a vertex marked as the initial vertex of the game (the initial city). Each player chooses a vertex of the graph, the first player chooses first, and the two players alternate their moves. At each move, the sequence of vertices chosen by the two players must form a simple path in the graph, starting from the distinguished initial vertex.

Player 1 wins the game if, after some number of moves, Player 2 has no valid move (that is no move that forms a simple path with the sequence of previous moves).

Generalized Geography (GG for short) is the following problem:

- INPUT: a directed graph  $G$  and an initial vertex  $s$ .
  - QUESTION: does Player 1 have a winning strategy for a GG game played on  $G$  from  $s$ ?
1. Show that GG is in PSPACE
  2. Exhibit a logarithmic space reduction  $f$  from QBF to GG. Carefully prove that the reduction is logspace and the equivalence  $w \in \text{QBF} \Leftrightarrow f(w) \in \text{GG}$ .
  3. What can you deduce about GG?