

# Advanced Complexity

TD n°4

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## Exercise 1: Language theory

Show that the following problems are PSPACE-complete :

1. NFA Universality :

— INPUT : a non-deterministic automaton  $A$  over alphabet  $\Sigma$

— QUESTION :  $\mathcal{L}(A) = \Sigma^*$  ?

Bonus : what is the complexity of this problem for a DFA ?

2. NFA Equivalence

— INPUT : two non-deterministic automata  $A_1$  and  $A_2$  over the same alphabet  $\Sigma$

— QUESTION :  $L(A_1) = L(A_2)$

Bonus : what is the complexity of this problem for a DFA ?

3. DFA Intersection Vacuity :

— INPUT : deterministic automata  $A_1, \dots, A_m$  for some  $m$

— QUESTION :  $\bigcap_{i=1}^m L(A_i) = \emptyset$  ?

## Exercise 2: Did you get padding ?

Show that if  $P = PSPACE$ , then  $EXPTIME = EXPSPACE$ .

## Exercise 3: Too fast !

Show that  $ATIME(\log n) \neq L$ .

## Exercise 4: Direct application

Show that  $EXPSPACE = AEXPTIME$ .

*Hint : You may use that if  $f$  is space-constructible, then :*

$$SPACE(poly(f(n))) = ATIME(poly(f(n)))$$

## Exercise 5: Closure under morphisms

Given a finite alphabet  $\Sigma$ , a function  $f : \Sigma^* \rightarrow \Sigma^*$  is a morphism if  $f(\Sigma) \subseteq \Sigma$  and for all  $a = a_1 \cdots a_n \in \Sigma^*$ ,  $f(a) = f(a_1) \cdots f(a_n)$  ( $f$  is uniquely determined by the value it takes on  $\Sigma$ ).

1. Show that NP is closed under morphisms, that is : for any language  $L \in NP$ , and any morphism  $f$  on the alphabet of  $L$ ,  $f(L) \in NP$ .

2. Show that if P is closed under morphisms, then  $P = NP$ .