Test Formulae Approach

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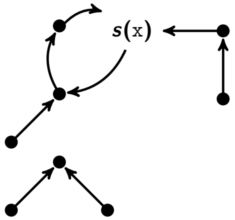
Memory states

A **memory state** is a pair (s, h) where:

- $s : \mathtt{VAR} \to \mathtt{LOC}$ is called store;
- $h : \mathtt{LOC} \to_{\mathsf{fin}} \mathtt{LOC}$ is called heap.

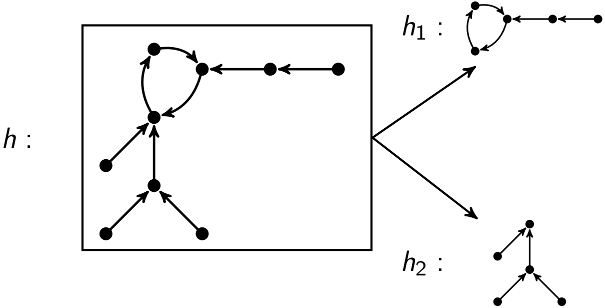
where $\mathtt{VAR} = \{\mathrm{x}, \mathrm{y}, \mathrm{z}, \dots\}$ set of (program) variables;
$\mathtt{LOC}$ set of locations (typically $\mathtt{LOC} \cong \mathbb{N} \cong \mathtt{VAR}$).



Generalisation:
h could be any finite graph

Note: Memory states are the standard model in Separation Logic

Splitting a Heap



$h = h_1 + h_2$ whenever

- $\mathrm{Dom}(h_1) \cap \mathrm{Dom}(h_2) = \emptyset$;
- h is the sum of the two functions h_1 and h_2.

What we want? To build Test Formulae

- Fix $\mathcal{X} \subseteq_{\text{fin}}$ $\mathtt{VAR}$ and let $n \in \mathbb{N}$;
- $\text{Test}_{\mathcal{X}}(n)$ definable finite set of sets of memory states
  - $\{(s,h) \mid$ in h there is a path from $s(x)$ to $s(y)\}, x, y \in \mathcal{X}$;
  - $\{(s,h) \mid h$ has a loop$\}$.

  or, equivalently $\text{Test}_{\mathcal{X}}(n)$ finite set of predicates and their semantics.

Indistinguishability relation $(s,h) \approx_n (s',h')$

- holds whenever $\forall T \in \text{Test}_{\mathcal{X}}(n)$, $(s,h) \in T \iff (s',h') \in T$;
- Property: for all $n, m \in \mathbb{N}$, if $m \geq n$ then $\approx_m \subseteq \approx_n$.

EF-style Game

Spoiler chose two structures (s, h) and (s', h'), and $n \in \mathbb{N}$ resources so that $(s, h) \approx_n (s', h')$. Then the games continue as follows:

- If $(s, h) \not\approx_n (s', h')$ then Spoiler wins;
- If $(s, h) \approx_n (s', h')$ and $n = 1$ then Duplicator wins;
- Otherwise,
  - Spoiler choses $n_1, n_2 \in \mathbb{N}$ so that $n = n_1 + n_2$ and two heaps h_1, h_2 so that $h = h_1 + h_2$;
  - Duplicator choses two heaps h'_1, h'_2 so that $h' = h'_1 + h'_2$;
  - Spoiler choses $i \in \{1, 2\}$. The game continues on the structures (s, h_i) and (s', h'_i), with n_i resources.

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Problem:
Given $\text{Test}_{\mathcal{X}}(1)$, find sufficient conditions on $\text{Test}_{\mathcal{X}}(n)$, for all $n \in \mathbb{N}$, so that Duplicator has a winning strategy.

Example: A family that works

Given $n \in \mathbb{N}$, let

- $\#\mathtt{loops}(\beta) \geq \beta'$ be the set

$$\{(s, h) \mid h \text{ with at least } \beta' \text{ loops of size } \beta \leq n\}$$

- $\#\mathtt{loops}^{\uparrow} \geq \beta'$ be the set

$$\{(s, h) \mid h \text{ with at least } \beta' \text{ loops of size } n + 1\}$$

- $\mathtt{garbage} \geq \beta$ the set

$$\{(s, h) \mid \text{ in Dom}(h) \text{ at least } \beta \text{ locations are not part of any loop}\}$$

Example: A family that works

Given $n \in \mathbb{N}$, let

- $\#\texttt{loops}(\beta) \geq \beta'$ be the set

Defining $\text{Test}_{\mathcal{X}}(n)$ as

$$\left\{ \begin{array}{l} \#\texttt{loops}(\beta) \geq \beta',\ \#\texttt{loops}^{\uparrow} \geq \beta', \\ \texttt{garbage} \geq \beta \end{array} \middle| \begin{array}{r} \beta \in [1, n] \\ \beta' \in \left[1, \frac{1}{2}n(n+3) - 1\right] \end{array} \right\}$$

Guarantees a strategy for Duplicator.

$\{(s, h) \mid$ in $\text{Dom}(h)$ at least β locations are not part of any loop$\}$