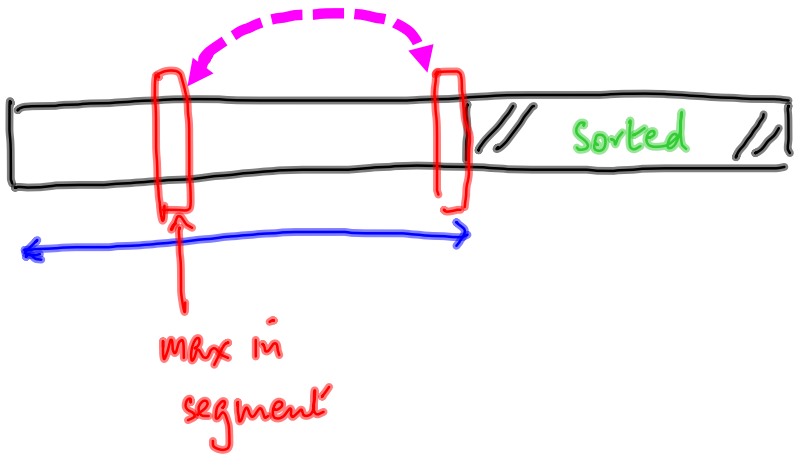
In place sorting

Selection Sort $O(n^2)$



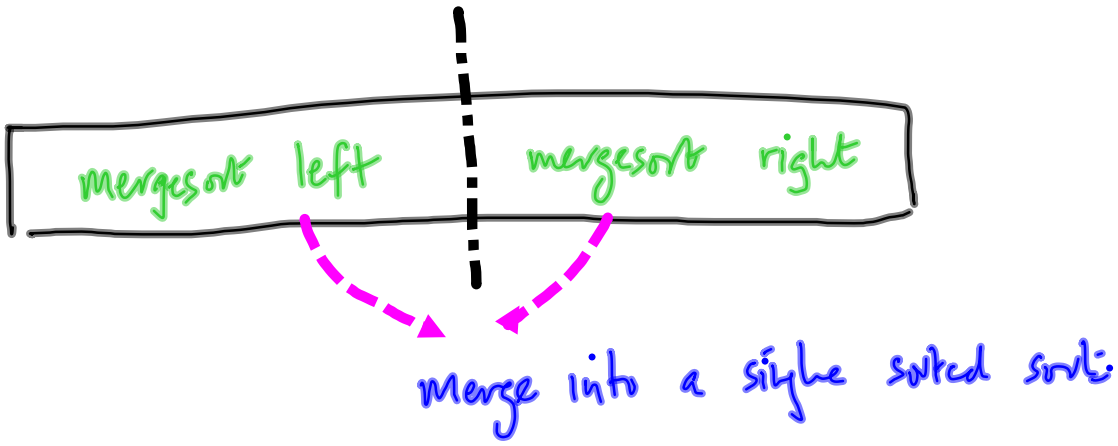
exchange max value with last value in unsorted segment

extend sorted suffix by one

More efficient sorting

Merge sort

Merging 2 sorted lists/arrays of length n takes O(n)



Complexity – set up & solve recurrence

$$T(n) = 2T(n/2) + \cancel{O(n)}\ n$$

2 recursive sorts merging

$$= 2\left(2T\left(\frac{n}{2 \cdot 2}\right) + \frac{n}{2}\right) + n$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2n$$

$$\vdots$$

$$= 2^k T\left(\frac{n}{2^k}\right) + kn$$

$T(0) = T(1) = 1$

At $k = \log_2 n$ $\frac{n}{2^k}$ becomes 1

$$T(n) = 2^{\log_2 n} T(1) + (\log_2 n) \cdot n$$

$$= n \cdot 1 + n \cdot \log n$$

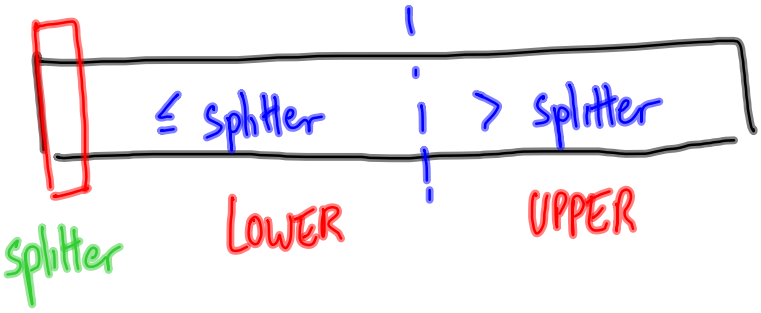
$$= O(n \log n)$$

n	10^3	10^6	Any standard CPU can do at most 10^9 operations/second
n^2	10^6	10^{12}	
$n \log_2 n$	10^4	$2 \cdot 10^7$	

For all practical purposes,

$O(n \log n)$ is as good as $O(n)$

Quicksort




```
qsort l = (qsort lower) ++ [splitter] ++ (qsort upper)
```

needs no merging

Good point : No merging required

Bad point : Split may be uneven

Worst Case recurrence

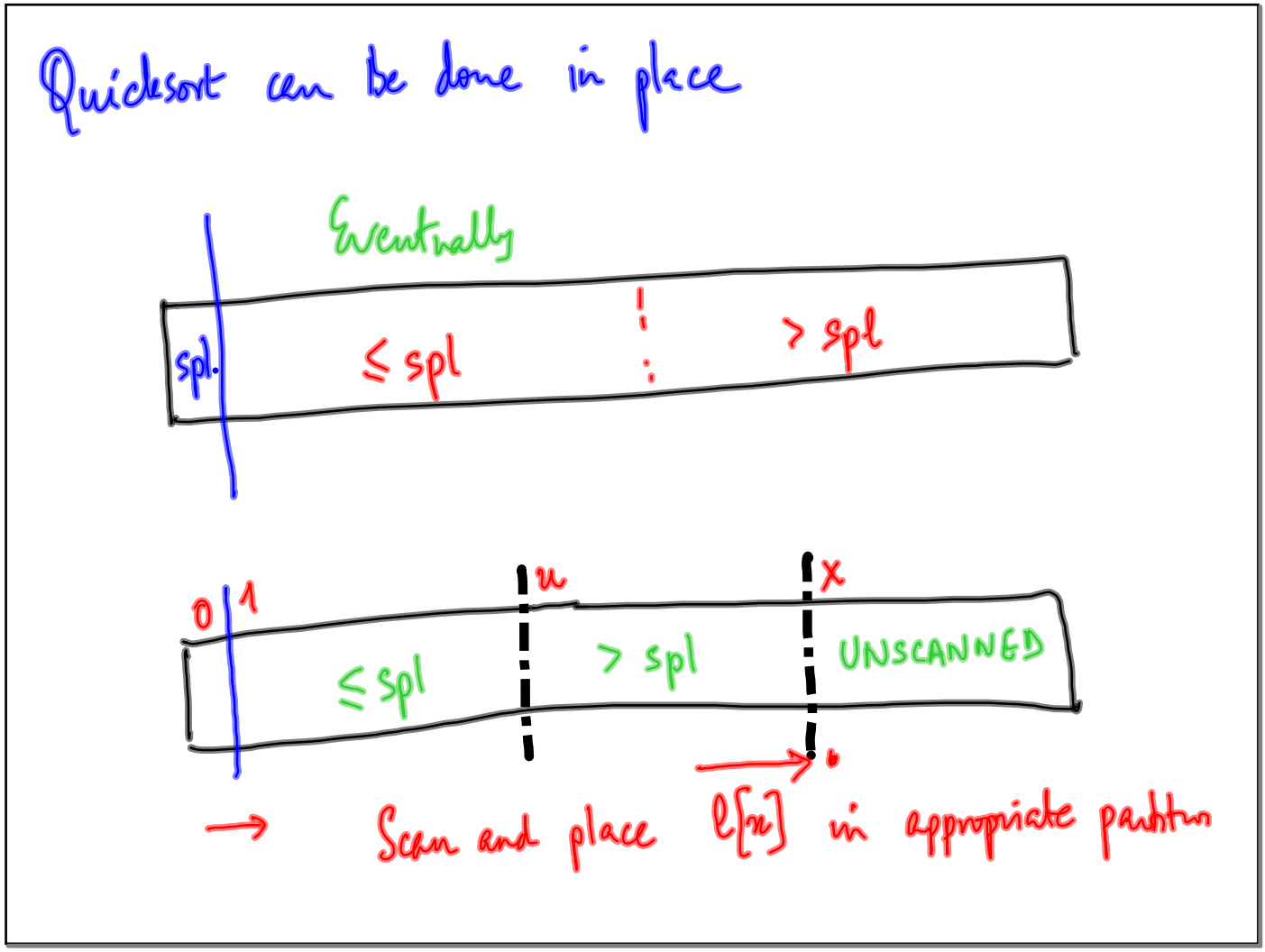
$$T(n) = T(n-1) + n$$

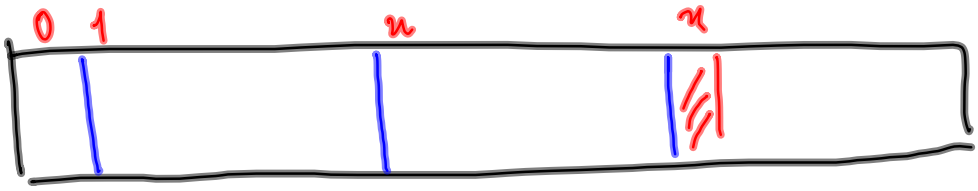
$$\vdots$$

$$n + n-1 + \cdots = O(n^2)$$

But on the average quicksort is $O(n \log n)$

randomly choose splitter – achieve this on the average

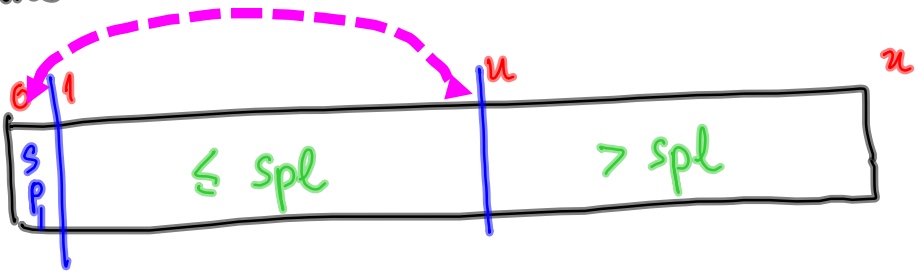

Quicksort can be done in place
Eventually
spl.
≤ spl
> spl
0
1
u
x
≤ spl
> spl
UNSCANNED
Scan and place l[x] in appropriate partition



$l[x] \leq$ splitter : {exchange $l[u]$, $l[x]$
increment u, x

$l[x] >$ splitter : increment x

After one scan



Want spl between lower & upper segments.

Exchange l[0], l[u-1]

To sort lower, upper in place

Add left, right boundaries as arguments

def qsort(l, left, right)

Can actually implement qsort as a loop, without recursion

Typically, built in sort functions use quicksort

Quicksort was invented by C.A.R. Hoare approx 50 years ago, before PL's had recursion; in fact, written in assembly lang

Mergesort & Quicksort are both examples of

Divide & Conquer

Netflix challenge

Movie rental shop

Customers rate movies

Make good recommendation

Identify users with similar preferences

2 users, N items, each user ranks N items

i : $i_1 <_i i_2 <_i \cdots <_i i_N$

j : $j_1 <_j j_2 <_j \cdots <_j j_N$

i & j agree on a pair of items x & y if the relative rankings are the same

$x <_i y$ iff $x <_j y$

Measuring similarity

How many of the $\frac{N \cdot (N-1)}{2}$ pairs do i & j agree on?

Fix an ordering of the N items as ranked by i

For i: $1 <_i 2 <_i 3 \ldots <_i N$

For j: $j_1 <_j j_2 <_j j_3 \ldots <_j j_N$

$(j_1, j_2, \ldots, j_N)$ is a permutation of $(1, 2, \ldots N)$

e.g. 3 2 1 4 5 vs 1 2 3 4 5

(3,2) is an inversion

For each position, check for smaller values to right

(3,2), (3,1) $O(n^2)$

(2,1)

=) 3 inversions (out of 10)

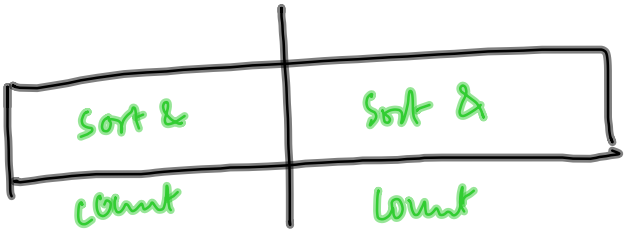
Can we do better

There could be $O(n^2)$ inversions

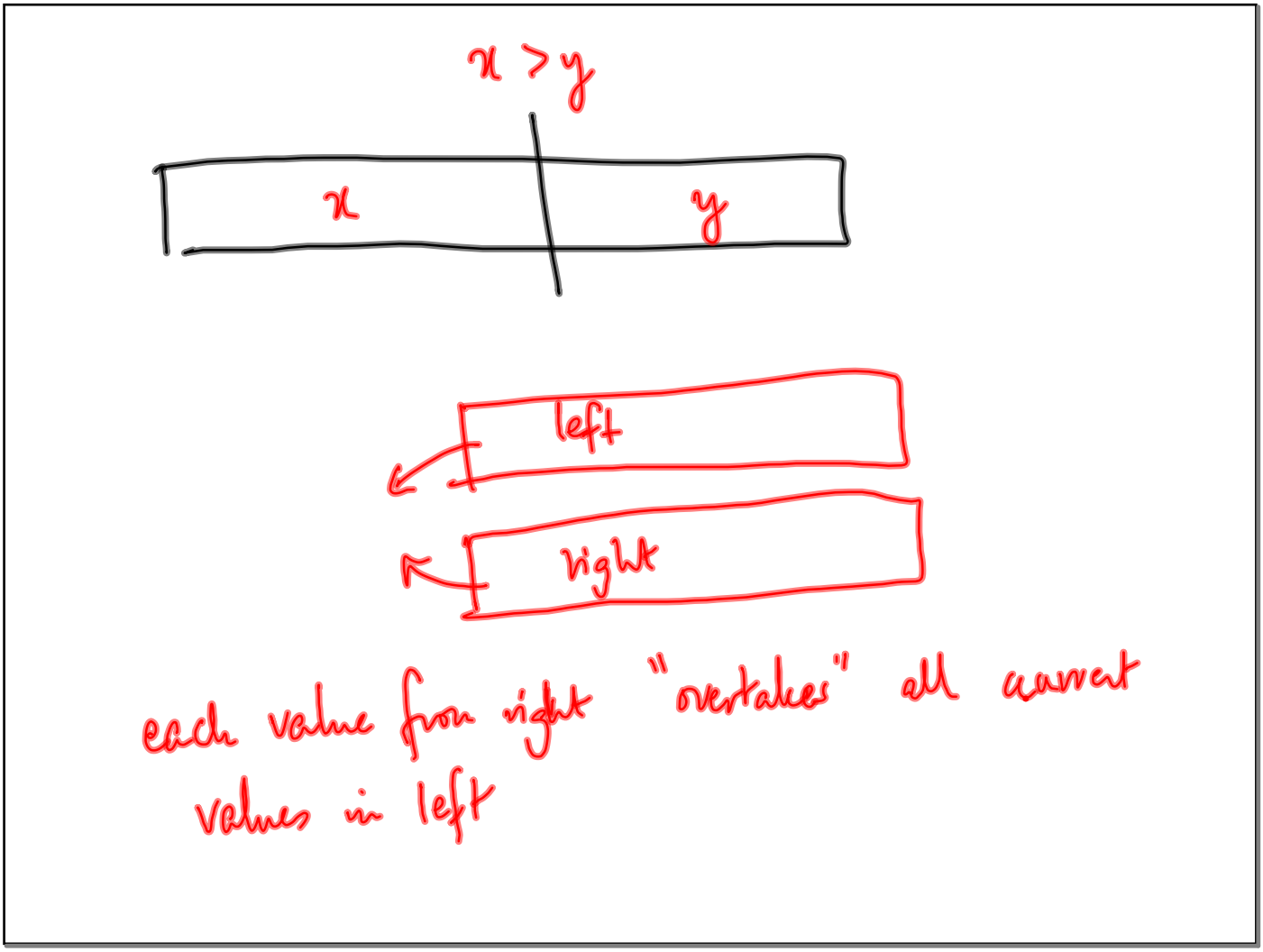
Can we count them without enumerating them?

Can we divide & conquer?

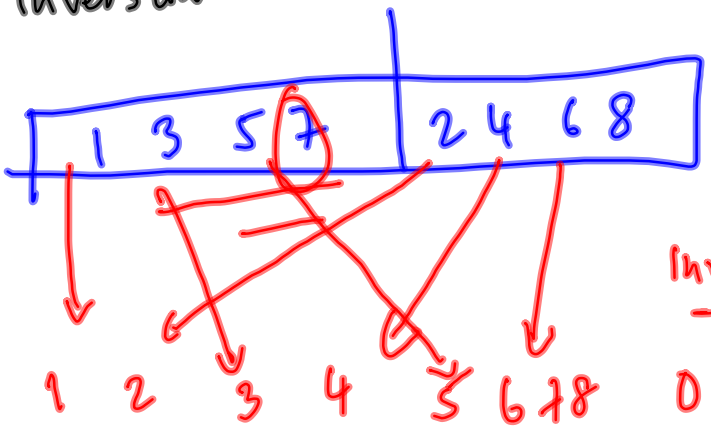
Like merge sort



Need to count inversions across boundary


x > y
x
y
left
right
each value from right "overtakes" all current values in left

In merge, whenever we add an element from right half, count current left & add to inversion



Inversion count

0 + 3 + 0 + 2 + 0 + 1 + 0 + 0

= 6 inversions