Chennai Mathematical Institute
Probability Theory: January-April 2014

Assignment 1

1. Let N denote the number of tigers in Corbett National Park. 15 tigers were captured, tagged, and released back into the population. After a period of six months, 8 tigers were captured. Find the probability that at least two of the tigers in the sample have tags. What assumptions are you making to compute this probability?

2. A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first.

3. Let $\mathcal{C}$ be a $\sigma - field$. Let $A \in \mathcal{C}$. Define

$$\mathcal{C}_{\mathcal{A}} = \{C; C = B \cap A \text{ for some } B \in \mathcal{C}\}.$$

   Show that $\mathcal{C}_{\mathcal{A}}$ is a $\sigma - field$. [Complements of sets are formed with respect to A]

4. The Hardy-Weinberg Law of Genetics: Each hereditary trait in an offspring depends on a pair of genes, one contributed by the father and the other by the mother. A gene is either recessive (denoted by $\boldsymbol{a}$) or dominant (denoted by $\boldsymbol{A}$). The hereditary trait is $\boldsymbol{A}$ if one gene in the pair is dominant ($\boldsymbol{AA}$, $\boldsymbol{Aa}$, $\boldsymbol{aA}$), and the trait is $\boldsymbol{a}$ if both genes in the pair are recessive ($\boldsymbol{aa}$). Suppose that the probabilities of the father carrying the pairs $\boldsymbol{AA}$, $\boldsymbol{Aa}$ (which is treated as the same as $\boldsymbol{aA}$), and $\boldsymbol{aa}$ are p_0 , q_0 , and r_0 , respectively, where $p_0 + q_0 + r_0 = 1$. The same probabilities hold for the mother. Also suppose that the gene from each parents is randomly inherited, so that each gene of a pair has a 50% chance of being passed on to the offspring.

   Assume that matings are random and the genetic contributions of the father and mother are independent.

   Show that the corresponding probabilities for a first-generation offspring are:

$$p_1 = (p_0 + q_0/2)^2; q_1 = 2(p_0 + q_0/2)(r_0 + q_0/2); r_1 = (r_0 + q_0/2)^2$$

5. Consider the two electrical circuits presented in Figure 1. Let A denote the event that switch A fails to close. Define similar events B, C, and D. Let E be the event that the circuit is completed. Express the event E in terms of A, B, C, and D using unions, intersections, and complements.