**Chennai Mathematical Institute**
**Probability Theory: January-April 2014**

**Assignment 3: Due March 13**

1. IQ as measured by a standard test is known to be normally distributed with a mean of 100 and a standard deviation of 15.

   (a) What is the probability that a randomly chosen individual will have an IQ less than 90?

   (b) An individual is classified as 'exceptional' if he or she falls in the top 1 %. What IQ score would determine the cut-off for an individual to be classified as exceptional.

2. The life of a certain brand of car tire has an exponential distribution with a mean of 25,000 km. Twenty tires from the manufacturing line are selected and tested. What is the probability that at least 15 tires in the sample survive beyond 30,000 kilometers? Clearly state the assumptions you are making.

3. A random variable $X$ is said to follow a Pareto distribution if it's pdf is given by

$$f_X(x) = \begin{cases} \frac{\beta\alpha^\beta}{x^{\beta+1}} & x \geq \alpha \\ 0 & x < \alpha \end{cases}$$

   where $\alpha$ and $\beta$ are positive.

   Show that the moments of order $r$ exist if and only if $r < \beta$.

4. Let $X$ be a random variable with the Rayleigh distribution, with density function given by

$$f(x|\beta) = \frac{1}{\beta^2} x e^{-\frac{1}{2}(x/\beta)^2}, \qquad x > 0$$

   where $\beta > 0$. Show that the mean and variance exist, and find them.

5. Let $X$ be a continuous random variable with pdf $f(x)$. The hazard function is the ratio of the probability density function to the survival function, $S(x) = 1 - F(x)$, i.e.

$$h(x) = \frac{f(x)}{1 - F(x)}.$$

   (a) Find the hazard function for the Weibull distribution:

$$f(x) = \frac{\gamma}{\beta} x^{\gamma-1} exp(-x^\gamma/\beta), \qquad x \geq 0.$$

   (b) Find the hazard function for the Gompertz distribution:

$$S(t) = exp\left[-\frac{e^\lambda}{\gamma}(e^{\gamma t} - 1)\right]$$

   (c) Express the cdf $F(x)$ in terms of the hazard function.