

Chennai Mathematical Institute

Probability Theory: Quiz III

Name \_\_\_\_\_

Answer all questions and show your work.

- (1)  $X$  and  $Y$  are continuous random variables with joint pdf given by

$$f(x, y) = \begin{cases} k(xy - x) & 0 < x < 1, 0 < y < 1 \\ 0 & o.w. \end{cases}$$

- (a) Find  $k$ .
  - (b) Find the marginal densities of  $X$  and  $Y$ .
  - (c) Find the conditional density of  $X$  given  $Y = y$ .
- (2) Let  $X_1, X_2$  be independent exponential random variables with parameter  $\beta$ . Find the distribution of  $V = X_1 + X_2$ . (You may use any method covered in class).
- (3) Consider the bivariate random vector  $(X, Y)$ , where  $X$  represents the concentration of a pollutant in parts per million (ppm) and  $Y$  represents the survival time in days of catfish exposed to the pollutant. We are given the following expression for the conditional expectation:

$$E(Y|X = x) = 42 - 0.2x.$$

- (a) If the average concentration is 80 ppm, find the average survival time.
- (b) Explain what the slope and intercept mean in this context.