

**Chennai Mathematical Institute**  
**Probability Theory: January-April 2014**

**Assignment 4: Due April 1**

1. Consider the bivariate joint distribution function given by

$$F(x, y) = 1 - e^{-x} - e^{-y} + e^{-x-y-\delta xy}, \quad x \geq 0, y \geq 0, 0 \leq \delta \leq 1.$$

Find the marginal pdf's of  $X$  and  $Y$ .

2. Let  $X$  and  $Y$  be independent Poisson random variables. Find the conditional distribution of  $X$  given  $X + Y$ .
3. An urn contains 5 red, 3 black, and 2 green marbles. Two marbles are selected from the urn without replacement. Let  $X$  denote the number of red marbles in the final selection and  $Y$  denote the number of black marbles in the final selection.
- (a) Find the joint pmf of  $X$  and  $Y$  and the marginals.
- (b) Find the conditional distributions.
- (c) Find the correlation coefficient.

4. Let  $(X, Y)$  have the **Bivariate Normal** distribution with joint pdf given by:

$$f(x, y) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \left( \frac{x-\mu_X}{\sigma} \right)^2 - 2\rho \left( \frac{x-\mu_X}{\sigma} \right) \left( \frac{y-\mu_Y}{\sigma} \right) + \left( \frac{y-\mu_Y}{\sigma} \right)^2 \right] \right\}$$

Here  $|\rho| < 1$ .

- (a) Find the conditional pdf of  $X$  given  $Y = y$ .
- (b) Find the joint pdf of  $U = X + Y$  and  $V = Y$ , and then the marginal distributions of  $U$  and  $V$ .
- (c) Find the contours of the bivariate normal: i.e. the locus where  $f(x, y)$  is constant. Show that the contours are circles when  $\rho = 0$ .