Advanced Complexity

TD n°4

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Exercise 1: Circuit value
We study the following variants of Circuit-Value, all the reductions are logspace in the following exercise.

1. Show that Horn Sat is \(AL\)-easy
   — INPUT : a set \(\phi\) of Horn clauses
   — QUESTION : is \(\phi\) satisfiable?
2. Show that Circuit Value is easier than Horn Sat
   — INPUT : a circuit \(C\) using \(\lor, \land, \lnot\)
   — QUESTION : does \(C\) evaluate to \(\top\)
3. Show that Monotone Circuit Value is \(AL\)-hard and easier than Circuit Value.
   — INPUT : a circuit \(C\) using \(\lor, \land\)
   — QUESTION : does \(C\) evaluate to \(\top\)
   
   Hint : build the configuration graph of the MT...
4. Show that Horn Sat is \(P\)-complete. Deduce that \(AL = P\).
   
   Hint : show that \(AL\) is stable under logspace reductions

Exercise 2: Language theory
Show that the following problems are \(PSPACE\)-complete:

1. NFA Universality
   — INPUT : a non-deterministic automaton \(A\) over alphabet \(\Sigma\)
   — QUESTION : \(L(A) = \Sigma^*\)?
   
   Bonus : what is the complexity of this problem for a DFA?
2. NFA Equivalence
   — INPUT : two non-deterministic automata \(A_1\) and \(A_2\) over the same alphabet \(\Sigma\)
   — QUESTION : \(L(A_1) = L(A_2)\)
   
   Bonus : what is the complexity of this problem for a DFA?
3. DFA Intersection Vacuity
   — INPUT : deterministic automata \(A_1, \ldots, A_m\) for some \(m\)
   — QUESTION : \(\bigcap_{i=1}^{m} L(A_i) = \emptyset\)?

Exercise 3: Too fast!
Show that \(ATIME(log \ n) \neq L\).

Exercise 4: Direct application
Show that \(EXPSPACE = AEXPTIME\).

Hint : You may use that if \(f\) is space-constructible, then :

\[ SPACE(poly(f(n)) = ATIME(poly(f(n))) \]