

# Transducers

## Session 8: Linear Regular Functions

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TA mail\* Course page<sup>†</sup> Exercises page<sup>‡</sup>

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### 1 Coding

**Exercise 1** (Using For-Transducers). Code the following functions

- $w \mapsto ww$
- $w \mapsto w_{\text{odd}}w_{\text{even}}$
- $wa \mapsto aw$
- $(ab)^n \mapsto a^n b^n$
- $xy \mapsto (xy)^{|w|}$
- $\mathbf{1}_L$  where  $L$  is a regular language
- $k$ -nested map-reverse
- $w \mapsto \prod_{i,j \text{ lex}} w_i w_j$

Describe the number of nested loops used.

**Exercise 2** (First order !). Let  $L$  be a first-order definable language. Write a monotone for-program that computes  $\mathbf{1}_L$ , i.e., a for-program where the only possible assignment of variables is  $x := \top$ .

**Exercise 3** (Polyblind). We define polyblind for-transducers are those where only the innermost variable loop can be tested against.

1. Prove that those programs do not compose.
2. Prove that the squaring function (without underscores) is polyblind.
3. Prove that the squaring function (with underscores) is not polyblind.
4. Prove that the function that maps  $i$  to  $\sum_{j \leq i} j$  is polyblind.
5. Prove that the function that maps  $i$  to  $\sum_{j \leq i} j$  can be realised by a monotone for-program.
6. Can the function  $i \mapsto \sum_{j \leq i} j$  be realised by a blind **and** monotone for-program?

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<sup>†</sup><https://www.mimuw.edu.pl/~bojan/2023-2024/przekształcenia-automatowe-transducers>

<sup>‡</sup><https://aliaumel.github.io/transducer-exercices/>

## A Hints