

Second Homework

Session 2: Polyregular Functions

Version: v0.0.10

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TA mail* Course page[†] Exercises page[‡]

June 24, 2024

- **Due:** 2024-06-03 18:15 CET
- **Organizer:** <mailto:ad.lopez@uw.edu.pl>
- **Summary:** MIMUW Transducer Course Homework 2
- **Url:** <https://www.irif.fr/~alopez/ressources/teaching/mimuw-2024/homework-2.pdf>

This homework is due for the 3rd of June, 18:15 GMT+1. It should be sent to ad.lopez@uw.edu.pl. Exercises are independent, and can be skipped without penalty. Failure to deliver the homework in due time is heavily penalized.

Please submit your homework in PDF format, and make sure to include your name and student identifier in the document. Every exercise should be present in your copy, if you fail to solve an exercise, please write I couldn't solve this exercise and move on to the next one.

The answers should be written in English. Please keep the answers concise and precise. Please separate statements from proofs in your answers, and provide a proof (even a short one) for every statement made. Handwaving proofs are not accepted. Proofs by intimidation are not accepted. Proofs by obfuscation are not accepted.

1 Mandatory Exercises

Exercise 1 (Invertible functions). Let $f: \Sigma^* \rightarrow \Gamma^*$ be an injective function. Is it true that if f is linear regular, then so is the partial inverse f^{-1} ? What about polyregular?

Exercise 2 (Context-free languages). Let L be a context-free language, and f be a polyregular function. Is it decidable whether $f^{-1}(L) \neq \emptyset$?

Exercise 3 (Image Intersection). Let f_1 and f_2 be two polyregular functions from Σ^* to Γ^* . Is it decidable whether $f_1(\Sigma^*) \cap f_2(\Sigma^*) = \emptyset$?

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[†]<https://www.mimuw.edu.pl/~bojan/2023-2024/przeznaczalacenia-automatowe-transducers>

[‡]<https://aliaumel.github.io/transducer-exercices/>

Exercise 4 (Image of linear regular). Let f be a linear regular function from Σ^* to Γ^* . We define $\text{growth}_f(n)$ to be the maximum among all words $w \in \Sigma^{\leq n}$ of the length of $f(w)$. Prove that one of the following holds:

1. The function growth_f is bounded.
2. There exists a $\alpha > 0$, for large enough $n \in \mathbb{N}$, $\text{growth}_f(n)$ is at least αn .

Exercise 5 (Polyregular Reductions). Consider two decision problems (which is the same thing as a language) $A \subseteq \Sigma^*$ and $B \subseteq \Gamma^*$. A polyregular reduction from a problem A to a problem B is a polyregular function $f: \Sigma^* \rightarrow \Gamma^*$ such that for all $w \in \Sigma^*$, $w \in A$ if and only if $f(w) \in B$.

We write boolean formulas as strings over the alphabet $\Sigma := \{a, \neg, \wedge, \vee, (,)\}$. Variables are encoded as a^n for some n . For instance, $(a \wedge \neg(aa \vee aaaa))$ is a valid formula which uses three variables a , aa and $aaaa$. The language of valid formulas is given by the following grammar: $F \mapsto (F \wedge F) \mid (F \vee F) \mid \neg F \mid V$ and $V \mapsto aV \mid a$. In particular, valid formulas are well-bracketed expressions.

We call **SAT** the list of satisfiable boolean formulas written over this alphabet. We call **CNFSAT** the list of satisfiable boolean formulas where the formula is in conjunctive normal form (CNF), that is, it is a conjunction of disjunctions of variables or their negations.

Construct a polyregular reduction from **SAT** to **CNFSAT**.

2 Optional Exercises x

The optional exercises are listed in the webpage <https://aliaumel.github.io/transducer-exercices/hall-of-fame.html>. There are plenty of exercises to choose from, and you can select any of them to solve. We divide the number of points given by the number of contestants that provided a solution, so choose wisely.

A Hints