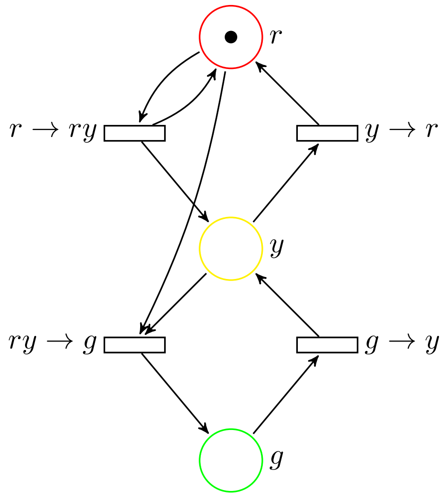
TD 9: Petri Nets

Exercise 1 (Traffic Lights)**.** Consider again the traffic lights example from the lecture notes:



1. How can you correct this Petri net to avert unwanted behaviours (like $r \to ry \to rr$) in a 1-safe manner?
2. Extend your Petri net to model two traffic lights handling a street intersection.

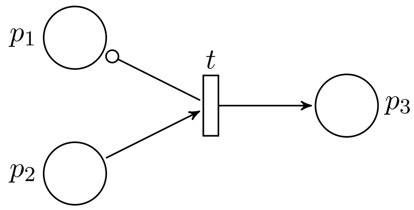
Exercise 2 (Producer/Consumer)**.** A producer/consumer system gathers two types of processes:

producers who can make the actions *produce* (p) or *deliver* (d), and

consumers with the actions *receive* (r) and *consume* (c).

All the producers and consumers communicate through a single unordered channel.

1. Model a producer/consumer system with two producers and three consumers. How can you modify this system to enforce a maximal capacity of ten simultaneous items in the channel?
2. An *inhibitor arc* between a place p and a transition t makes t firable only if the current marking at p is zero. In the following example, there is such an inhibitor arc between p_1 and t. A marking $(0, 2, 1)$ allows to fire t to reach $(0, 1, 2)$, but $(1, 1, 1)$ does not allow to fire t.



Using inhibitor arcs, enforce a priority for the first producer and the first consumer on the channel: the other processes can use the channel only if it is not currently used by the first producer and the first consumer.

Exercise 3 (Model Checking Petri Nets)**.** Let us fix a Petri net $\mathcal{N} = \langle P, T, F, W, m_0 \rangle$. We consider as usual propositional LTL, with a set of atomic propositions AP equal to P the set of places of the Petri net. We define proposition p to hold in a marking m in $\mathbb{N}^P$ if $m(p) > 0$.

The models of our LTL formulæ are *computations* $m_0 m_1 \cdots$ in $(\mathbb{N}^P)^\omega$ such that, for all $i \in \mathbb{N}$, $m_i \rightarrow_{\mathcal{N}} m_{i+1}$ is a transition step of the Petri net $\mathcal{N}$.

1. We want to prove that state-based LTL model checking can be performed in polynomial space for 1-safe Petri nets. For this, prove that one can construct an exponential-sized Büchi automaton $\mathcal{B}_{\mathcal{N}}$ from a 1-safe Petri net that recognizes all the infinite computations of $\mathcal{N}$ starting in m_0.

2. In the general case, state-based LTL model checking is undecidable. Prove it for Petri nets with at least two unbounded places, by a reduction from the halting problem for 2-counter Minsky machines.

3. We consider now a different set of atomic propositions, such that $\Sigma = 2^{\mathrm{AP}}$, and a labeled Petri net, with a labeling homomorphism $\lambda : T \to \Sigma$. The models of our LTL formulæ are infinite words $a_0 a_1 \cdots$ in Σ^ω such that $m_0 \xrightarrow{t_0}_{\mathcal{N}} m_1 \xrightarrow{t_1}_{\mathcal{N}} m_2 \cdots$ is an execution of $\mathcal{N}$ and $\lambda(t_i) = a_i$ for all i.

   Prove that *action-based* LTL model checking can be performed in polynomial space for labeled 1-safe Petri nets.

Exercise 4 (VASS)**.** An n-dimensional *vector addition system with states* (VASS) is a tuple $\mathcal{V} = \langle Q, \delta, q_0 \rangle$ where Q is a finite set of states, $q_0 \in Q$ the initial state, and $\delta \subseteq Q \times \mathbb{Z}^n \times Q$ the transition relation. A configuration of $\mathcal{V}$ is a pair (q, v) in $Q \times \mathbb{N}^n$. An execution of $\mathcal{V}$ is a sequence of configurations $(q_0, v_0)(q_1, v_1) \cdots (q_m, v_m)$ such that $v_0 = \bar{0}$, and for $0 < i \leq m$, $(q_{i-1}, v_i - v_{i-1}, q_i)$ is in δ.

1. Show that any VASS can be simulated by a Petri net.

2. Show that, conversely, any Petri net can be simulated by a VASS.

Exercise 5 (VAS)**.** An n-dimensional *vector addition system* (VAS) is a pair $\langle v_0, W\rangle$ where $v_0 \in \mathbb{N}^n$ is the initial vector and $W \subseteq \mathbb{Z}^n$ is the set of transition vectors. An execution of (v_0, W) is a sequence $v_0 v_1 \cdots v_m$ where $v_i \in \mathbb{N}$ for all $0 \leq i \leq m$ and $v_i - v_{i-1} \in W$ for all $0 < i \leq m$.

We want to show that any n-dimensional VASS $\mathcal{V} = \langle Q, \delta, q_0\rangle$ can be simulated by an $(n+3)$-dimensional VAS $\langle v_0, W\rangle$. Let $k = |Q|$, and $q_0, \ldots, q_{k-1}$ the states of $\mathcal{V}$. We define two functions $a(i) = i+1$ and $b(i) = (k+1)(k-i)$. We encode a configuration (q_i, v) of $\mathcal{V}$ as the vector $(v(1), \ldots, v(n), a(i), b(i), 0)$. For every state q_i, $0 \leq i < k$, we add two transition vectors to W:

$$t_i = (0, \ldots, 0, -a(i), a(k-1-i) - b(i), b(k-1-i))$$
$$t'_i = (0, \ldots, 0, b(i), -a(k-1-i), a(i) - b(k-1-i))$$

For every transition $d = (q_i, w, q_j)$ of $\mathcal{V}$, we add one transition vector to W:

$$t_d = (w(1), \ldots, w(n), a(j) - b(i), b(j), -a(i))$$

1. Show that any execution of $\mathcal{V}$ can be simulated by (v_0, W) for a suitable v_0.

2. Conversely, show that this VAS (v_0, W) simulates $\mathcal{V}$ faithfully.