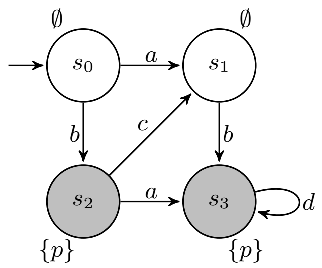


Figure 1: A Kripke structure.

TD 6: Partial-Order Reduction Solutions

Exercise 1

1. $I = \{(a,b),(a,d),(b,c),(b,d),(c,d)\}$ closed by symmetry. Only $(a,c) \notin I$ due to states s_1 and s_6. $U = \{a,c\}$.

2.
   - $red(s_0) = en(s_0) = \{b,d\}$ and $red(s_7) = en(s_7) = \{b,d\}$ (there is no invisible action from either s_0 or s_7).
   - $red(s_3) = en(s_3) = \{b\}$ and $red(s_4) = en(s_4) = \{b\}$ to satisfy (C0).
   - Because a and c depend on each other, $red(s_1) = en(s_1) = \{a,c\}$ and $red(s_6) = en(s_6) = \{a,c\}$.
   - Finally, $red(s_2) = en(s_2) = \{a,c,d\}$ and $red(s_5) = en(s_5) = \{a,c,d\}$ because we can't remove a or c and keep the other withouth breaking condition (C1), can't remove both because d is not invisible and can't remove anything if we keep c because it creates a cycle with only s_2 (resp. s_5).

Exercise 2

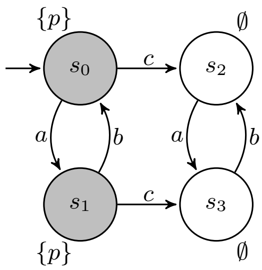
1. Suppose we have some function red satisfying $(C1)$. Consider a state s and suppose there exists a in $red(s)$ and b in $en(s) \backslash red(s)$ such that a and b are not independant from each other. Then there exists t such that $s \xrightarrow{b} t$ in $\mathcal{K}$ and the path $s \xrightarrow{b} t$ violates (C1): this is a contradiction.

2. Consider the Kripke structure described in Figure 1, with $red(s) = en(s)$ for $s \neq s_0$ and $red(s_0) = \{a\}$. This satisfies (C0),(C1'),(C2),(C3) because a and b are independent from each other and a is invisible. One is able to differentiate between the original Kripke structure and its reduction thanks to a formula such as $\neg p \mathsf{U} (p \mathsf{U} \neg p)$, which is satisfied by the run $s_0 \to s_2 \to s_1 \to s_3$.

Exercise 3

1. (C0), (C2) and (C3) are trivially satisfied. (C1) is satisfied as well because a and b on one side, a and c on the other side are independant from each other.
2. $\mathsf{E}(\top \,\mathsf{U}\, (\mathsf{E}(p \,\mathsf{U}\, q) \wedge \mathsf{E}(p \,\mathsf{U}\, (\neg p \wedge \neg q))))$ cannot be satisfied if s_7 cannot be reached.
3. $red(s_0) = a$, and for $s \neq s_0$ $red(s) = en(s)$ works.

Exercise 4



In the Kripke structure defined above, the assignments $red(s_0) = \{a\}$, $red(s_1) = \{b\}$, $red(s_2) = \{a\}$ and $red(s3) = b$ satisfy (C0)-(C2) but results in a Kripke structure where states s_2 and s_3 are unattainable.