Homework 5

To hand in on October 26 (Wednesday) in my office or by email at leroux@lsv.fr. Answers can be written in french or in english.

Exercise 1. Let $AP = \{p, q\}$, and $\Sigma = 2^{AP} = \{a, b, c, d\}$, where $a = \{p\}$, $b = \{q\}$, $c = \{p, q\}$, and $d = \emptyset$. We identify each letter in $\Sigma$ with a boolean formula over $AP$, for instance, $a = p \land \neg q$.

For each LTL formula $\phi$ below, give a Büchi automaton acccepting the language $L(\phi) = \{w \in \Sigma^\omega \mid w, 0 \models \phi\}$.

1. $(G F a) \rightarrow (G F b)$
2. $G(a \rightarrow (\neg a SU b))$
3. $G(X b \rightarrow a)$
4. $(G F a) \land (F b) \land (F c)$

Exercise 2. A Büchi automaton $A = (Q, \Sigma, I, T, F)$ is deterministic if $|I| \leq 1$, and for each state $q$ in $Q$ and symbol $a$ in $\Sigma$, $|(q, a, q') \in T \mid q' \in Q| \leq 1$.

1. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by deterministic Büchi automata, are closed under intersection.

2. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by deterministic Büchi automata, are closed under union.