

## Homework 5

To hand in on October 26 (Wednesday) in my office or by email at [leroux@lsv.fr](mailto:leroux@lsv.fr).

Answers can be written in french or in english.

**Exercise 1.** Let  $AP = \{p, q\}$ , and  $\Sigma = 2^{AP} = \{a, b, c, d\}$ , where  $a = \{p\}$ ,  $b = \{q\}$ ,  $c = \{p, q\}$ , and  $d = \emptyset$ . We identify each letter in  $\Sigma$  with a boolean formula over  $AP$ , for instance,  $a = p \wedge \neg q$ .

For each LTL formula  $\varphi$  below, give a Büchi automaton accepting the language  $L(\varphi) = \{w \in \Sigma^\omega \mid w, 0 \models \varphi\}$ .

1.  $(GF a) \rightarrow (GF b)$
2.  $G(a \rightarrow (\neg a SU b))$
3.  $G(Xb \rightarrow a)$
4.  $(GF a) \wedge (Fb) \wedge (Fc)$

**Exercise 2.** A Büchi automaton  $\mathcal{A} = (Q, \Sigma, I, T, F)$  is *deterministic* if  $|I| \leq 1$ , and for each state  $q$  in  $Q$  and symbol  $a$  in  $\Sigma$ ,  $|\{(q, a, q') \in T \mid q' \in Q\}| \leq 1$ .

1. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by *deterministic* Büchi automata, are closed under intersection.
2. Show that the set of languages recognizable by Büchi automata, and the set of languages recognizable by *deterministic* Büchi automata, are closed under union.