Homework 2

To hand in on October 7th at the beginning of the exercise session, or by email at `leroux@lsv.fr`.

Exercise 1 (Equivalences)**.** We fix a set of atomic propositions AP including $\{p, q\}$, and the time flow $(\mathbb{N}, <)$. Wich of the following equivalences are correct? Give a proof or a counter-example.

1. $(\mathsf{X}\, p) \wedge (\mathsf{X}\, q) \equiv \mathsf{X}(p \wedge q)$
2. $(\mathsf{SF}\, p) \wedge (\mathsf{SF}\, q) \equiv \mathsf{SF}(p \wedge q)$
3. $(\mathsf{Y}\, p)\ \mathsf{S}\ (\mathsf{Y}\, q) \equiv \mathsf{Y}(p\ \mathsf{S}\ q)$
4. $(\mathsf{G}\, p)\ \mathsf{U}\ (\mathsf{G}\, q) \equiv \mathsf{G}(p\ \mathsf{U}\ \mathsf{G}\, q)$
5. $(\mathsf{X}\, p)\ \mathsf{U}\ q \equiv \mathsf{X}(p\ \mathsf{U}\ (p \wedge q))$
6. $(p\ \mathsf{U}\ q)\ \mathsf{U}\ q \equiv p\ \mathsf{U}\ q$
7. $(\mathsf{G}\, \mathsf{F}\, p) \rightarrow (\mathsf{G}\, \mathsf{F}\, q) \equiv \mathsf{G}(p \rightarrow \mathsf{F}\, q)$
8. $\mathsf{G}\, p \rightarrow \mathsf{F}\, q \equiv p\ \mathsf{U}\ (q \vee \neg p)$

Exercise 2 (Specification)**.** We fix a set of propositions $\mathrm{AP} = \{\mathrm{ok}, \mathrm{crash}, \mathrm{alarm}, \mathrm{reset}\}$ and the time flow $(\mathbb{N}, <)$. Provide formulæ for the following properties (a) in $\mathrm{FO}(\mathrm{AP}, <)$, (b) in $\mathrm{TL}(\mathrm{AP}, \mathsf{SU}, \mathsf{SS})$ (your formula should use past modalities), and (c) in $\mathrm{TL}(\mathrm{AP}, \mathsf{SU})$. For cases (b) and (c), your formula φ must be such that a temporal structure w satisfies the property described if and only if $w, 0 \models \varphi$.

1. "Whenever the alarm rings, there has been a crash immediately before."
2. "Whenever the alarm rings, there has been a crash some time before, and no reset in the meantime."