Homework 8

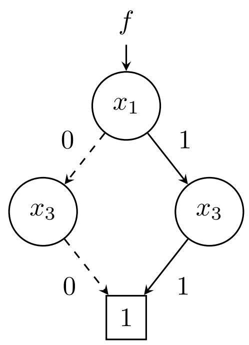
To hand in on December 9th at the beginning of the exercise session, or by email (before 14:00) at `leroux@lsv.fr`.

Answers can be written in french or in english.

Exercise 1. Draw the BDDs for the following functions, using the order of your choice on the variables $\{x_1, x_2, x_3\}$. You may omit the 0-node. No justification is necessary.

1. $(x_1 \Leftrightarrow x_2) \vee (x_1 \Leftrightarrow x_3)$,
2. $s(x_1, x_2, x_3) = \begin{cases} 1 & \text{if } x_1 \texttt{ xor } x_2 \texttt{ xor } x_3 = 1 \\ 0 & \text{otherwise}. \end{cases}$

Exercise 2. Let $x_1, \ldots, x_n$, be Boolean variables, for some $n \geq 1$. We fix the ordering $x_1 < \cdots < x_n$. Given a function f, we let $B(f)$ denote the number of nodes labelled with variables in the BDD for f. For instance, the figure below shows the BDD of $f := x_1 \Leftrightarrow x_3$, where we have $B(f) = 3$.



Depending on n, how many different functions f exist such that

1. $B(f) = 1$?
2. $B(f) = 2$?
3. $B(f) = 3$?