Simple Priced Timed Games are not that simple

From FSTTCS 2015

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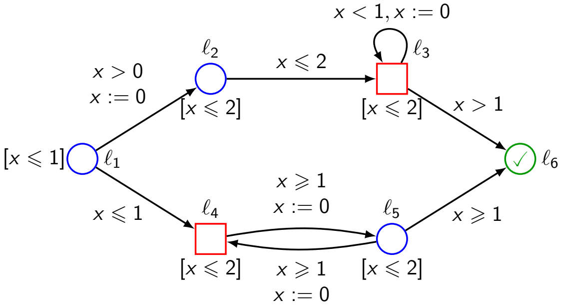
September 7, 2016

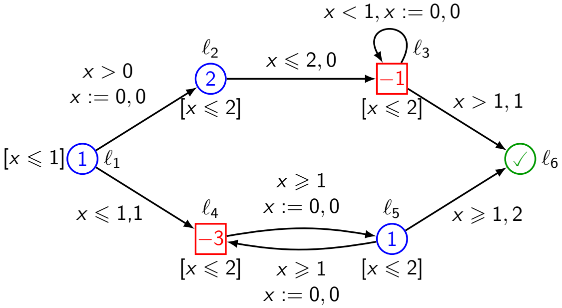
Priced Timed Games



Timed Automaton
with partition of states
between 2 players
+ reachability objective
+ rates in locations
+ costs over transitions

Semantics in terms of
infinite game with weights

Priced Timed Games



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$(\ell_1, 0)$

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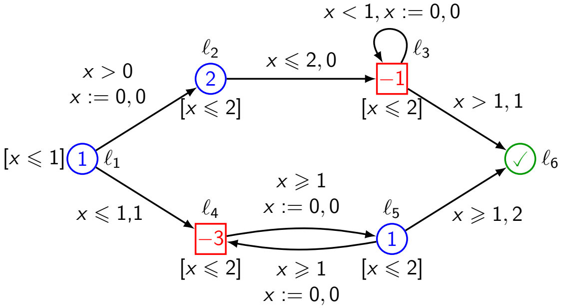

$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4)$

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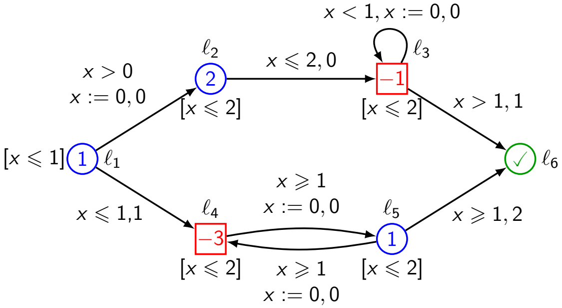

$(\ell_1, 0) \xrightarrow{0.4, \searrow} (\ell_4, 0.4) \xrightarrow{0.6, \rightarrow} (\ell_5, 0)$

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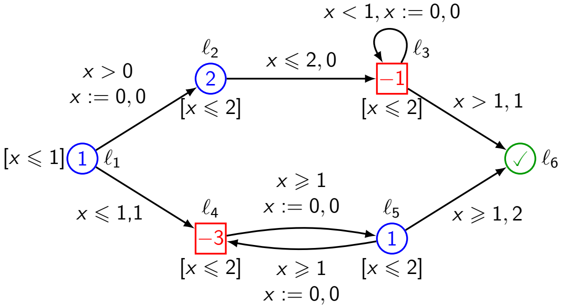

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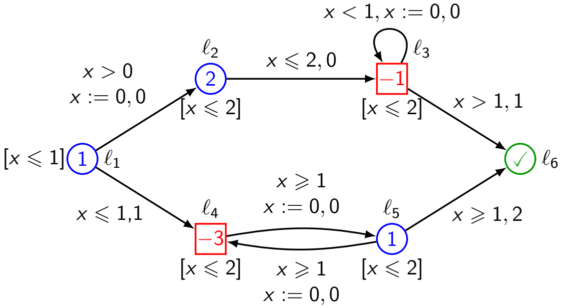

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$$0.4 + 1 \quad -3 \times 0.6 \quad +1.5 \quad -3 \times 1.1 \quad +2 \times 2 + 2 \quad = 3.8$$

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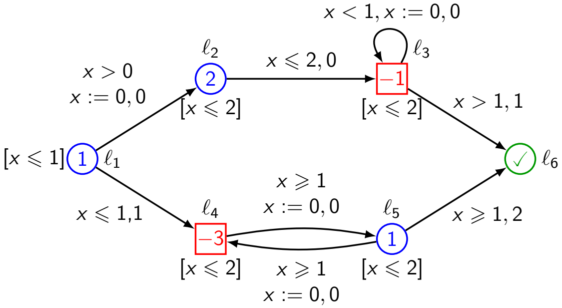

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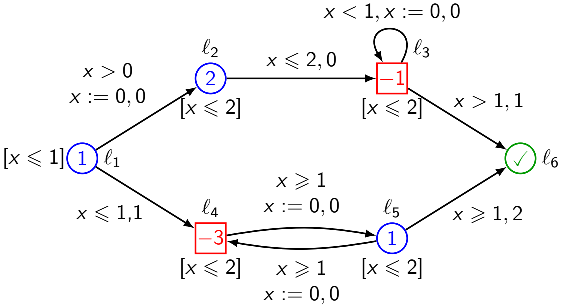
$$(\ell_1, 0) \xrightarrow{0.2, \nearrow} (\ell_2, 0) \xrightarrow{0.9, \rightarrow} (\ell_3, 0.9) \xrightarrow{0.2, \circlearrowleft} (\ell_3, 0) \xrightarrow{0.9, \circlearrowleft} (\ell_3, 0) \quad \cdots$$

$$0.2 \quad +0.9 \quad -0.2 \quad -0.9 \quad \cdots \quad = +\infty \ (\checkmark \text{ not reached})$$

Priced Timed Games



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$$(\ell_1, 0) \xrightarrow[0.4+1]{0.4, \searrow} (\ell_4, 0.4) \xrightarrow[-3 \times 0.6]{0.6, \rightarrow} (\ell_5, 0) \xrightarrow[+1.5]{1.5, \leftarrow} (\ell_4, 0) \xrightarrow[-3 \times 1.1]{1.1, \rightarrow} (\ell_5, 0) \xrightarrow[+2 \times 2 + 2]{2, \nearrow} (\checkmark, 2) \quad = 3.8$$

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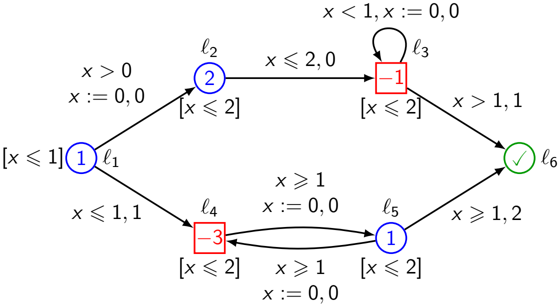
$$\text{Cost of a play: } \begin{cases} +\infty & \text{if } \checkmark \text{ not reached} \\ \text{total payoff up to } \checkmark & \text{otherwise} \end{cases}$$

Strategies and objectives



Strategy for each player: mapping of finite runs to a delay and an action

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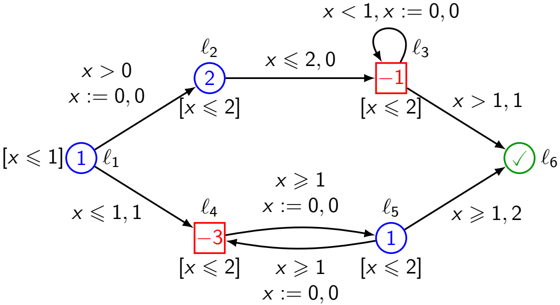

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Goal of player ○: reach ✓ **and** minimize the cost

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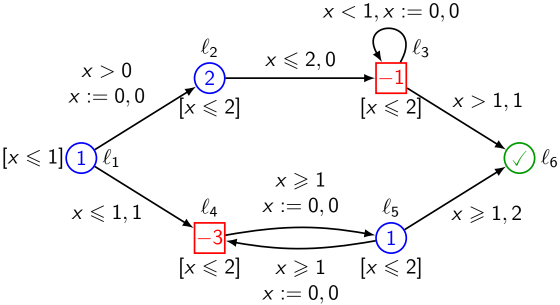

Strategy for each player: mapping of finite runs to a delay and an action

Goal of player ○: reach ✓ **and** minimize the cost
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Main object of interest:
$\overline{\mathsf{Val}}(\ell, v)$ = minimal cost player ○ can guarantee
$\underline{\mathsf{Val}}(\ell, v)$ = maximal cost player □ can guarantee
What can players guarantee as a payoff? And design *good* strategies.

State of the art

$\mathsf{F}_{\leqslant K}\checkmark$: $\exists$ a strategy in the PTG (priced timed game) for player $\bigcirc$ reaching $\checkmark$ with a cost $\leqslant K$?

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- 2-player PTGs: undecidable [Brihaye, Bruyère, and Raskin, 2005, Bouyer, Brihaye, and Markey, 2006a], even with only non-negative costs and 3 clocks
- PTGs with non-negative costs and strictly non-Zeno cost cycles or with one clock: exponential algorithm [Bouyer, Cassez, Fleury, and Larsen, 2004, Alur, Bernadsky, and Madhusudan, 2004, Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011, Hansen, Ibsen-Jensen, and Miltersen, 2013]

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- $\mathsf{F}_{\leqslant K}$✓ undecidable for 2 or more clocks and pseudo-polynomial algorithm for One-clock Bi-valued PTG [Brihaye, Geeraerts, Krishna, Manasa, Monmege, and Trivedi, 2014]

One-clock Bi-valued PTG: important restriction on the allowed rates of locations

Inspired by other previous techniques for 1-clock PTGs?

[Hansen, Ibsen-Jensen, and Miltersen, 2013]: strategy improvement algorithm

[Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011]: iterative elimination of locations

- precomputation: polynomial-time cascade of simplification of 1-clock PTGs into simple 1-clock PTGs (SPTGs)
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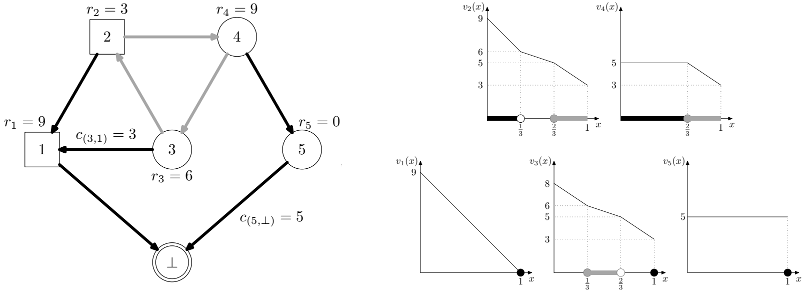
- for SPTGs: compute value functions $\overline{\mathsf{Val}}(\ell, x)$.



Recursive elimination of states

Intuition from [Bouyer, Larsen, Markey, and Rasmussen, 2006b, Rutkowski, 2011]:

- Player ○ prefers to stay as long as possible in locations with **minimal price**: add a final location allowing him to stay until the end, and make the location urgent

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Challenges with arbitrary weights:

- Proof of correctness does not generalise: initially two distinct proofs for ○ and □
- Proof of termination does not generalise: difficult because of the double recursion...

Make a symmetric treatment of ○ and □

Theorem

PTGs are determined ($\overline{\mathsf{Val}} = \underline{\mathsf{Val}}$), and value functions are continuous (over regions).

Determinacy follows from Gale-Stewart determinacy result.

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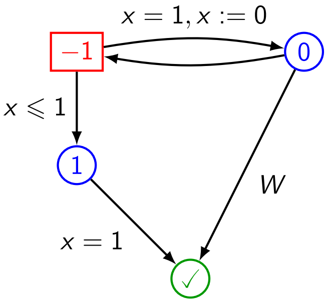
For general 1-clock PTGs?

- removing guards and invariants: previously used techniques work!
- removing resets: previously, bound the number of resets...

Bounding the number of resets needed is not possible



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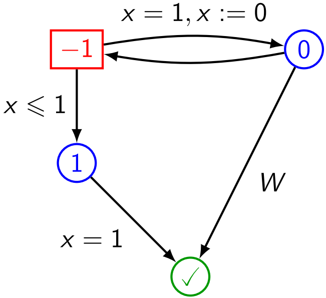

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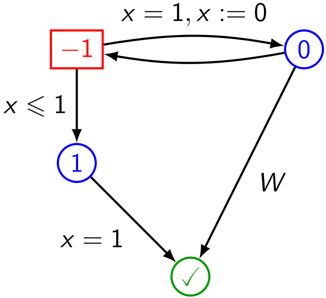

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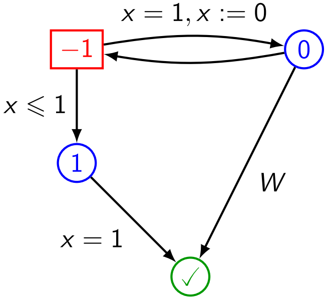

Player ○ can guarantee (i.e., ensure to be below) value ε for all $\varepsilon > 0$...

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... moreover, to obtain ε, ○ needs to loop at least $W + \lceil 1/\log \varepsilon \rceil$ times before reaching ✓!

Current solution: Reset-acyclic 1-clock PTGs

exponential time algorithm for reset-acyclic 1-clock PTGs with arbitrary weights

Summary and Future Work

Results

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Thank you for your attention

References I

Rajeev Alur, Mikhail Bernadsky, and P. Madhusudan. Optimal reachability for weighted timed games. In *Proceedings of the 31st International Colloquium on Automata, Languages and Programming (ICALP'04)*, volume 3142 of *Lecture Notes in Computer Science*, pages 122–133. Springer, 2004.

Patricia Bouyer, Franck Cassez, Emmanuel Fleury, and Kim G. Larsen. Optimal strategies in priced timed game automata. In *Proceedings of the 24th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'04)*, volume 3328 of *Lecture Notes in Computer Science*, pages 148–160. Springer, 2004.

Patricia Bouyer, Thomas Brihaye, and Nicolas Markey. Improved undecidability results on weighted timed automata. *Information Processing Letters*, 98(5):188–194, 2006a.

Patricia Bouyer, Kim G. Larsen, Nicolas Markey, and Jacob Illum Rasmussen. Almost optimal strategies in one-clock priced timed games. In *Proceedings of the 26th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'06)*, volume 4337 of *Lecture Notes in Computer Science*, pages 345–356. Springer, 2006b.

Thomas Brihaye, Véronique Bruyère, and Jean-François Raskin. On optimal timed strategies. In *Proceedings of the Third international conference on Formal Modeling and Analysis of Timed Systems (FORMATS'05)*, volume 3829 of *Lecture Notes in Computer Science*, pages 49–64. Springer, 2005.

References II

Thomas Brihaye, Gilles Geeraerts, Shankara Narayanan Krishna, Lakshmi Manasa, Benjamin Monmege, and Ashutosh Trivedi. Adding Negative Prices to Priced Timed Games. In *Proceedings of the 25th International Conference on Concurrency Theory (CONCUR'13)*, volume 8704 of *Lecture Notes in Computer Science*, pages 560–575. Springer, 2014.

Thomas Dueholm Hansen, Rasmus Ibsen-Jensen, and Peter Bro Miltersen. A faster algorithm for solving one-clock priced timed games. In *Proceedings of the 24th International Conference on Concurrency Theory (CONCUR'13)*, volume 8052 of *Lecture Notes in Computer Science*, pages 531–545. Springer, 2013.

Michał Rutkowski. Two-player reachability-price games on single-clock timed automata. In *Proceedings of the Ninth Workshop on Quantitative Aspects of Programming Languages (QAPL'11)*, volume 57 of *Electronic Proceedings in Theoretical Computer Science*, pages 31–46, 2011.