Submission date : 02.10.2019
Submission style : You may submit your HW by one of the following ways :

- Hand-written in English and submitted to me during the T.D.
- LaTeX file + PDF in French, sent to me by email.
- LaTeX file + PDF in English, sent to me by email - which will get you an extra 1 point bonus(where the maximum is 20).

Email : khmelnitsky@lsv.fr

Solve the following exercise :

Exercice 1 :
Show that $\forall k \in \mathbb{N}, \forall (n_1, n_2, \ldots, n_k) \in \mathbb{N}^k, \exists N \in \mathbb{N}$ such that, for any k (edge) coloring of the complete graph K_N, there exists a color $c \in [\![1, k]\!]$ for which there is a complete sub-graph K_{n_c} which is monochromatic in the color c.
(the smallest N for which this property holds is denoted by $R(n_1, \ldots, n_k)$).

Exercice 2 (Cantor's set) :
Let $I = [a, b]$ an interval of length $\delta = b - a$. Let $\rho(I) = [a, a + \frac{\delta}{3}] \cup [b - \frac{\delta}{3}, b]$. ρ cuts the interval I in to three closed intervals of equal length and removes interior of the middle one. We extend ρ to closed unions of disjoint closed intervals by making ρ act on each of the intervals separately.
Cantor set is $C = \cap_{n \in \mathbb{N}} F_n$, where F_n is defined recursively on n :

$$F_0 = [0, 1] \text{ and } F_{n+1} = \rho(F_n)$$

1. Show that C is closed, non-empty and of "length" 0(We mean the length of the sum of the intervals contained in it (but what we really want to say is the measure :))).
2. Let $n \in \mathbb{N}$. Show that F_n is a union of 2^n intervals $\cup_{i=1}^{2^n} [a_i, b_i]$. The ordered sequence $a_0, a_1, \ldots, a_{2^n}$ where the items are of the form $\frac{\sum_{i=1}^{n} x_i 3^i}{3^n}$, where $x_i \in \{0, 2\}$ (elements of $[0, 1]$ in base 3 which have at most n digits and constitute of 0 and 2) and $b_i = a_i + \frac{1}{3^n}$.
3. Deduce that C consists of all the numbers in $[0, 1]$ whose base 3 representation has only 0's and 2's.
4. Build a bijection from C to $[0, 1]$, using (3).
5. Deduce that C is uncountable.