Automates d'arbre

TD n°2 : Decision problems & tree homomorphisms

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Exercise 1 : Recognizing an abstract language.

1) Let $\mathcal{E}$ be a finite set of linear terms on $T(\mathcal{F}, \mathcal{X})$. Prove that $Red(\mathcal{E}) = \big\{C[t\sigma] \mid C \in \mathcal{C}(\mathcal{F}), t \in \mathcal{E}, \sigma \text{ ground substitution}\big\}$ is recognizable.
2) Prove that if $\mathcal{E}$ contains only ground terms, then one can construct a DFTA recognizing $Red(\mathcal{E})$ whose number of states is at most $n+2$, where n is the number of nodes of $\mathcal{E}$.

Exercise 2 : Decisions problems

We consider the **(GII)** problem (ground instance intersection) :

Instance : t a term in $T(\mathcal{F}, \mathcal{X})$ and $\mathcal{A}$ a NFTA

Question : Is there at least one ground instance of t accepted by $\mathcal{A}$?

1) Suppose that t is linear. Prove that **(GII)** is P-complete.
2) Suppose that $\mathcal{A}$ is deterministic. Prove that **(GII)** is NP-complete.
3) Prove that **(GII)** is EXPTIME-complete.
   hint : for the hardness, reduce the intersection non-emptiness problem (admitted to be EXPTIME-complete).
4) Deduce that the complement problem :
   Instance : t a term in $T(\mathcal{F}, \mathcal{X})$ and linear terms t_1, ..., t_n
   Question : Is there a ground instance of t which is not an instance of any t_i ?
   is decidable.

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Bonus exercise : Direct images of an homomorphism

Let $\mathcal{F} = \{f/2, g/1, a\}$ and $\mathcal{F}' = \{f'/2, g/1, a\}$. Let us consider the tree homomorphism h determined by h_F defined by : $h_{\mathcal{F}}(f) = f'(x_1, x_2), h_{\mathcal{F}}(g) = f'(x_1, x_1)$, and $h_{\mathcal{F}}(a) = a$.

1. Is $h(\mathcal{T}(\mathcal{F}))$ recognizable ?
2. Let$L_1 = \{g^i(a) | i \geq 0\}$, then L_1 is a recognizable tree language, is $h(L_1)$ recognizable ?