Exercise 1: First constructions of Tree Automatas

Let $F = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down DFTA for the set $G(t)$ of ground instances of the term $t = f(f(a,x), g(y))$ which is defined by:

$$G(t) = \{f(f(a,u), g(v)) \mid u, v \in T(F)\}$$

Exercise 2: What is recognizable by an FTA?

Are the following tree languages recognizable (by a bottom-up FTA)?

- $F = \{g(1), a(0)\}$ and $L$ the set of ground terms of even height.
- $F = \{f(2), g(1), a(0)\}$ and $L$ the set of ground terms of even height.

Exercise 3: Bottom-up vs Top-down

1) Recall why bottom-up NFTAs, bottom-up DTAs and top-down NFTAs have the same expressiveness.
2) Let $F = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down NFTA for the set $M(t)$ of terms which have a ground instance of the term $t = f(a,g(x))$ as a subterm, i.e. $M(t) = \{C[f(a,g(u))] \mid C \in C(F), u \in T(F)\}$.
3) Show that NFTAs and top-down DFTAs do not have the same expressiveness.

**Bonus exercise: Satisfiability**

Let $F = \{\text{and}(2), \text{or}(2), \text{not}(1), 0(0), 1(0), x(0)\}$. A ground term over $F$ can then be viewed as a boolean formula over $x$.

1) Give an NFTA which recognizes the set of satisfiable boolean formulae over $x$.

Let $F = \{\text{and}(2), \text{or}(2), \text{not}(1), 0(0), 1(0), x_1(0), \ldots, x_n(0)\}$, i.e. we now handle $n$ variables instead of a single one. The same variable may appear several times in a formula, and should be evaluated consistently.

2) Give an NFTA which recognizes the set of satisfiable boolean formulae over $x_1, \ldots, x_n$. 