Automates d'arbre

TD n°6 : PDL

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Exercise 1 : Warm up PDL

Definition 1 (PDL)
The syntax is the following :

$$\phi ::= a \mid \top \mid \neg\phi \mid \phi \vee \phi \mid \langle \pi \rangle\, \phi \qquad\qquad \text{(position formulae)}$$

$$\pi ::= \downarrow \mid \rightarrow \mid \pi^{-1} \mid \pi;\pi \mid \pi + \pi \mid \pi^* \mid \phi? \qquad\qquad \text{(path formulae)}$$
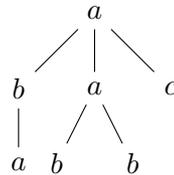
The semantic is defined this way : let t be a tree, we define $\llbracket\phi\rrbracket_t$ (resp. $\llbracket\pi\rrbracket_t$) as a set of positions of t (resp. a relation on positions of t) by induction on the size of ϕ (resp. π) :

$$\llbracket a \rrbracket_t = \{w \in Pos(t) \mid t(w) = a\} \qquad\qquad \llbracket \downarrow \rrbracket_t = \{(w, w.i) \mid w, w.i \in Pos(t)\}$$

$$\llbracket \top \rrbracket_t = Pos(t) \qquad\qquad \llbracket \rightarrow \rrbracket_t = \{(w.i, w.(i+1)) \mid w.i, w.(i+1) \in Pos(t)\}$$

$$\llbracket \neg\phi \rrbracket_t = Pos(t) \setminus \llbracket \phi \rrbracket_t \qquad\qquad \llbracket \pi^{-1} \rrbracket_t = \llbracket \pi \rrbracket_t^{-1}$$

$$\llbracket \phi_1 \vee \phi_2 \rrbracket_t = \llbracket \phi_1 \rrbracket_t \cup \llbracket \phi_2 \rrbracket_t \qquad\qquad \llbracket \pi_1;\pi_2 \rrbracket_t = \llbracket \pi_2 \rrbracket_t \circ \llbracket \pi_1 \rrbracket_t$$

$$\llbracket \langle \pi \rangle\, \phi \rrbracket_t = \llbracket \pi \rrbracket_t^{-1}(\llbracket \phi \rrbracket_t) \qquad\qquad \llbracket \pi_1 + \pi_2 \rrbracket_t = \llbracket \pi_1 \rrbracket_t \cup \llbracket \pi_2 \rrbracket_t$$

$$\llbracket \pi^* \rrbracket_t = \llbracket \pi \rrbracket_t^* \qquad\qquad \llbracket \phi? \rrbracket_t = \Delta_{\llbracket\phi\rrbracket_t} = \{(w,w) \mid w \in \llbracket \phi \rrbracket_t\}$$

Let t be a tree and $w, w' \in Pos(t)$. We note :
— $t, w \models \phi$ *if* $w \in \llbracket \phi \rrbracket_t$
— $t \models \phi$ *if* $t, \epsilon \models \phi$ *and we say that t satisfies ϕ*
— $t, w, w' \models \pi$ *if* $(w, w') \in \llbracket \pi \rrbracket_t$

Let t be the tree :

Which formulae are satisfied by t ?

1. $\phi_1 = \neg a \vee \langle \downarrow \rangle \left(\neg \langle \leftarrow \rangle \top \wedge b \wedge \langle \rightarrow^* \rangle (c \wedge \neg \langle \rightarrow \rangle \top) \right)$

2. $\phi_2 = \neg a \vee \langle \downarrow \rangle \left(\neg \langle \leftarrow \rangle \top \wedge b \wedge \langle (\rightarrow; c?)^* \rangle (\neg \langle \rightarrow \rangle \top) \right)$

3. $\phi_3 = \langle (a?; \downarrow)^* \rangle (a \wedge \neg \langle \downarrow \rangle \top)$

Exercise 2 : The power of PDL ?

Give a translation of PDL in MSO which preserves models. That is, given a position formula ϕ (resp. a path formula π), construct a MSO formula $\tilde{\phi}$ (resp. $\tilde{\pi}$) whose set of free variable is $\{X_a \mid a \in \mathcal{F}\} \cup \{x\}$ (resp. $\{X_a \mid a \in \mathcal{F}\} \cup \{x, y\}$) such that $t, w \models \phi$ iff $(P_a(t))_{a \in \mathcal{F}}, w \models \tilde{\phi}$ (resp. $t, w, w' \models \pi$ iff $(P_a(t))_{a \in \mathcal{F}}, w, w' \models \tilde{\pi}$) where $P_a(t) = \{w \in Pos(t) \mid t(w) = a\}$.

Exercise 3 : The exerice we won't have time for

Fix an alphabet $\mathcal{F}$. Give a PDL formula π such that :

- for all tree t and all position p of t, there exists exactly one position q of t such that $(p, q) \in [\![\pi]\!]_t$ (π defines a function on positions).
- for all tree t and position p of t, $(p, q) \in [\![\pi^*]\!]_t$ iff q is a position of t such that $t(q) = t(p)$.