Automates d'arbre

TD n°4 : Logic and Hedges

Charlie Jacomme

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Exercise 1 : MSO on finite trees

We consider trees with maximum arity 2. Give MSO formulae which express the following :

1. X is closed under predecessors
2. $x \subseteq y$ (with $\subseteq$ the prefix relation on positions)
3. 'a' occurs twice on the same path
4. 'a' occurs twice not on the same path
5. There exists a sub tree with only a's
6. The frontier word contains the chain 'ab'

Exercise 2 : From formulaes to automaton

Give tree automatons recognizing the languages on trees of maximum arity 2 defined by the formulae :

1. $(x \in S \wedge (x \downarrow_1 y \Rightarrow y \in S)) \wedge (z \in S \Rightarrow P_f(z))$
2. $\exists S.(x \in S \wedge (x \downarrow_1 y \Rightarrow y \in S)) \wedge (z \in S \Rightarrow P_f(z))$

Exercise 3 : The power of Wsks

Produce formulae of WSkS for the following predicates :

- the set X has exactly two elements.
- the set X contains at least one string beginning with a 1.
- $x \leq_{lex} y$ where $\leq_{lex}$ is the lexicographic order on $\{1, ...k\}^*$.
- given a formula of WSkS ϕ with one free first-order variable, produce a formula of WSkS expressing that there is an infinity of words on $\{1, ..., k\}^*$ satisfying ϕ.

Exercise 4 : The limit of Wsks

Prove that the predicate $x = 1y$ is not definable in WSkS.

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Homework for next week : To the infinity...

Let $\Sigma = \{a, b\}$. Define a DFHA $\mathcal{A}$ such that $L(\mathcal{A})$ is the set of all trees such that "for every leaf labeled with a, there is an ancestor from which there is a path whose nodes are labeled with b". Here "ancestor" means strict ancestor and "from which there is a path" means that there is a path from a son of this ancestor to a leaf.