Automates d'arbre

TD n°3 : Minimization and Logic

Exercise 1 : Minimization results

Definition 1 *An equivalence relation $\equiv$ on T is a congruence on $T(\mathcal{F})$ if for every $f \in \mathcal{F}_n$:*

$$(\forall i, 1 \leq i \leq n, u_i \equiv v_i) \Rightarrow f(u_1, ..., u_n) \equiv f(v_1, ..., v_n)$$

For a given tree language L, let us define the congruence $\equiv_L$ on $T(\mathcal{F})$ by : $u \equiv_L v$ if for all contexts $C \in C(\mathcal{F})$:

$$C[u] \in L \Leftrightarrow C[v] \in L$$

Prove that the following are equivalent :

1. L is a recognizable tree language
2. L is the union of some equivalence classes of a congruence of finite index
3. the relation $\equiv_L$ is a congruence of finite index. Then, show how to obtain the minimal automaton of a language.

Exercise 2 : Let's try to minimize

We consider the complete DFTA on $\mathcal{F} = \{f/2, g/2, a/0, b/0\}$ with states $\{q_a, q_b, q_f, q_g, \top, \bot\}$, finale state $\top$ and transitions :

— $a \longrightarrow q_a$
— $b \longrightarrow q_b$
— $f(q_a, q_b) \longrightarrow q_f$
— $f(q_f, q_b) \longrightarrow \top$
— $g(q_a, q_a) \longrightarrow q_q$
— $f(q_g, q_b) \longrightarrow \top$
— $h(q, q') \longrightarrow \top$ if $h \in \{f, g\}$, and $q = \top$ or $q' = \top$.
— $h(q, q') \longrightarrow \bot$ in all other cases.

Give the corresponding minimized algorithm obtained through the partition refinement algorithm.

Exercise 3 : MSO on finite trees

We consider trees with maximum arity 2. Give MSO formulae which express the following :

1. X is closed under predecessors
2. $x \subseteq y$ (with $\subseteq$ the prefix relation on positions)
3. 'a' occurs twice on the same path