Automates d’arbre

TD n°2 : Decision problems & tree homomorphisms

September 25, 2018

Exercise 1 : Recognizing an abstract language.
1) Let $E$ be a finite set of linear terms on $T(\mathcal{F}, \mathcal{X})$. Prove that $\text{Red}(E) = \{C[t\sigma] \mid C \in C(\mathcal{F}), t \in E, \sigma \text{ ground substitution}\}$ is recognizable.
2) Prove that if $E$ contains only ground terms, then one can construct a DFTA recognizing $\text{Red}(E)$ whose number of states is at most $n + 2$, where $n$ is the number of nodes of $E$.

Exercise 2 : Decisions problems
We consider the (GII) problem (ground instance intersection) :

Instance : $t$ a term in $T(\mathcal{F}, \mathcal{X})$ and $A$ a NFTA

Question : Is there at least one ground instance of $t$ accepted by $A$ ?
1) Suppose that $t$ is linear. Prove that (GII) is P-complete.
2) Suppose that $A$ is deterministic. Prove that (GII) is NP-complete.
3) Prove that (GII) is EXPTIME-complete.
   hint : for the hardness, reduce the intersection non-emptiness problem (admitted to be EXPTIME-complete).
4) Deducce that the complement problem :
   Instance : $t$ a term in $T(\mathcal{F}, \mathcal{X})$ and linear terms $t_1, \ldots, t_n$
   Question : Is there a ground instance of $t$ which is not an instance of any $t_i$ ?
   is decidable.

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Homework for next week : Direct images of an homomorphism
Let $\mathcal{F} = \{f/2, g/1, a\}$ and $\mathcal{F}' = \{f'/2, g/1, a\}$. Let us consider the tree homomorphism $h$ determined by $h_\mathcal{F}$ defined by : $h_\mathcal{F}(f) = f'(x_1, x_2), h_\mathcal{F}(g) = f'(x_1, x_1)$, and $h_\mathcal{F}(a) = a$.
1. Is $h(T(\mathcal{F}))$ recognizable ?
2. Let $L_1 = \{g^i(a) \mid i \geq 0\}$, then $L_1$ is a recognizable tree language, is $h(L_1)$ recognizable ?