Exercise 1: First constructions of Tree Automatas

Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down DFTA for the set $G(t)$ of ground instances of the term $t = f(f(a, x), g(y))$ which is defined by:

$$G(t) = \{f(f(a, u), g(v)) \mid u, v \in T(\mathcal{F})\}$$

Exercise 2: What is recognizable by an FTA?

Are the following tree languages recognizable (by a bottom-up FTA)?

- $\mathcal{F} = \{g(1), a(0)\}$ and $L$ the set of ground terms of even height.
- $\mathcal{F} = \{f(2), g(1), a(0)\}$ and $L$ the set of ground terms of even height.

Exercise 3: Bottom-up vs Top-down

1) Recall why bottom-up NFTAs, bottom-up DTAs and top-down NFTAs have the same expressiveness.
2) Let $\mathcal{F} = \{f(2), g(1), a(0)\}$. Give a DFTA and a top-down NFTA for the set $M(t)$ of terms which have a ground instance of the term $t = f(a, g(x))$ as a subterm, ie. $M(t) = \{C[f(a, g(u))] \mid C \in \mathcal{C}(\mathcal{F}), u \in T(\mathcal{F})\}$.
3) Show that NFTAs and top-down DFTAs do not have the same expressiveness.