Exercise 1: Polylogarithmic space
Let \( \text{polyL} = \bigcup_{k \in \mathbb{N}} \text{SPACE}(\log^k(n)) \). Show that \( \text{polyL} \neq \text{P} \).

Exercise 2: Padding argument
1. Show that if \( \text{DSPACE}(n^c) \subseteq \text{NP} \) for some \( c > 0 \), then \( \text{PSPACE} \subseteq \text{NP} \).
2. Deduce that \( \text{DSPACE}(n^c) \neq \text{NP} \).

Exercise 3: My very first PSPACE-complete problem
Show that the following problem is PSPACE-complete (not assuming anything about QBF):
— INPUT: a Turing Machine \( M \) and a word \( w \) and a number \( t \) written in unary
— QUESTION: does \( M \) accepts \( w \) within space \( t \)?

Exercise 4: PSPACE and games
The Geography game is played as follow:
— The game starts with a given name of a city, for instance Cachan;
— the first player gives the name of a city whose first letter coincides with the last letter of the previous city, for instance Nice;
— the second player gives then another city name, always starting with the last letter of the previous city, for instance Evry;
— the first player plays again, and so on – with the restriction that no player is allowed to give the name of a city already used in the game;
— the loser is the first player who does not find a new city name to continue.
This game can be described using a directed graph whose vertices represent cities and where an edge \((X,Y)\) means that the last letter of the city \( X \) is the same as the first letter of the city \( Y \). This graph has also a vertex marked as the initial vertex of the game (the initial city).
Each player choses a vertex of the graph, the first player choses first, and the two players alternate their moves. At each move, the sequence of vertices chosen by the two players must form a simple path in the graph, starting from the distinguished initial vertex.
Player 1 wins the game if, after some number of moves, Player 2 has no valid move (that is no move that forms a simple path with the sequence of previous moves).

GEOGRAPHY is the following problem:
— INPUT: a directed graph \( G \) and an initial vertex \( s \).
— QUESTION: is the player 1 sure to win the game on \( G \) starting at \( s \) ?
Show that GEOGRAPHY is PSPACE-complete by:
1. Showing that GEOGRAPHY is PSPACE
2. That the satisfiability of a QSAT formula of the form \( \exists x_1 \forall x_2 \ldots \exists x_n \wedge (\lor) \) can be expressed as a GEOGRAPHY instance.