

Advanced Complexity

TD n°1 : SPACE and NL

Charlie Jacomme

September 13, 2017

Exercise 1 : Graph representation and why it does not matter

Let $\Sigma = \{0, 1, ;, \bullet\}$, $n \in \mathbb{N}$ and $V = [0, n - 1]$. We consider the following two representations of a directed graph $G = (V, E)$ by a word in Σ^* :

- By its adjacency matrix : $m_{0,0}m_{0,1} \dots m_{0,n-1} \bullet \dots \bullet m_{n-1,0} \dots m_{n-1,n-1}$, where for all $i, j \in [0, n - 1]$, $m_{i,j}$ is equal to 1 if $(i, j) \in E$, 0 otherwise.
- By its adjacency list : $k_0^0; \dots; k_{m_1}^0 \bullet \dots \bullet k_0^{n-1}; \dots; k_{m_{n-1}}^{n-1}$, where for all i , $[k_0^i, \dots, k_{m_i}^i]$ is the list of neighbors of vertex i , written in binary, in increasing order.

1. Describe a logarithmic space bounded deterministic Turing machine which takes as input the graph G , represented by adjacency lists, and returns the adjacency matrix representation of G .
2. Conversely, describe a logarithmic space bounded deterministic Turing machine taking as input a graph G , represented by its adjacency matrix, and computing the adjacency list representation of G .

Therefore, the complexity of the problem REACH seen in class does not depend on the representation of the graph.

Exercise 2 : Inclusions of complexity classes

Definition 1. A function $f : \mathbb{N} \rightarrow \mathbb{N}$ is said to be space-constructible if $\forall n \in \mathbb{N} f(n) > \log(n)$ and there exists a deterministic Turing machine that computes $f(|x|)$ in $O(f(|x|))$ space given x as input.

Show that for a space-constructible function,

$$\text{NSPACE}(f(n)) \subseteq \text{DTIME}(2^{O(f(n))} + O(n))$$

Exercise 3 : Restrictions in the definition of $\text{SPACE}(f(n))$, and why they do not matter

In the course, we restricted our attention to Turing machines that always halt, and whose computations are space-bounded on every input. In particular, remember that $\text{SPACE}(f(n))$ is defined as the class of languages L for which there exists some deterministic Turing machine M that always halts (i.e. on every input), whose computations are $f(n)$ space-bounded (on every input), such that M decides L .

Now, consider the following two classes of languages :

- $\text{SPACE}'(f(n))$ is the class of languages L such that there exists a deterministic Turing machine M , running in space bounded by $f(n)$, such that M accepts x iff $x \in L$. Note that if $x \notin L$, M may not terminate.
- $\text{SPACE}''(f(n))$ is the class of languages L such that there exists a deterministic Turing machine M such that M accepts x using space bounded by $f(n)$ iff $x \in L$ (M may use more space and not even halt when $x \notin L$).

1. Show that for a space-constructible function $f = \Omega(\log n)$, $\text{SPACE}'(f(n)) = \text{SPACE}(f(n))$
2. Show that for a space-constructible function $f = \Omega(\log n)$, $\text{SPACE}''(f(n)) = \text{SPACE}(f(n))$

Exercise 4: Dyck's language —

Let A be the language of balanced parentheses – that is the language generated by the grammar $S \rightarrow (S) | SS | \epsilon$. Show that $A \in L$.

- What about the language B of balanced parentheses of two types? that is the language generated by the grammar $S \rightarrow (S) | [S] | SS | \epsilon$