Automates d'arbre

TD n°5 : Hedges and Alternation

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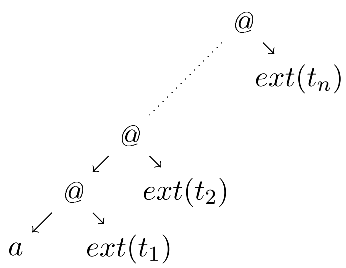
Exercise 1 : Extensions

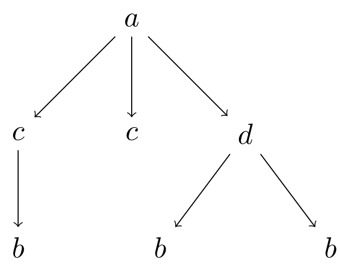
Definition 1 (extension encoding)
Let t be an unranked tree on Σ. Let $\mathcal{F}_{ext}^{\Sigma} = \{@(2)\} \cup \{a(0) \mid a \in \Sigma\}$. We define the ranked tree $ext(t)$ by induction on the size of t by :

- *for $a \in \Sigma$, $ext(a) = a$*
- *if $t = a(t_1, ..., t_n)$ with $n \geq 1$, $ext(t) = @(ext(a(t_1, ..., t_{n-1})), ext(t_n))$*

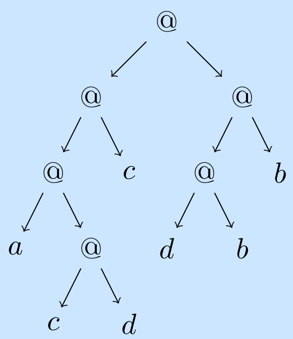
that is $ext(a(t_1, ..., t_n))$ is equal to :



Give the extension encoding of :



Solution:



Exercise 2 : The soundess of the extension

Let L be a language of unranked trees. Prove that L is recognizable by a NFHA iff $ext(L)$ is recognizable by a NFTA.

Solution:

$\Rightarrow$) Let $\mathcal{A} = \langle Q, \Sigma, \Delta, F\rangle$ be a NFHA recognizing L such that there is exactly one rule of the form $a(L_{a,q}) \longrightarrow q$ for all (a,q) and let $B_{a,q} = \langle P_{a,q}, Q, p^0_{a,q}, \delta_{a,q}, F_{a,q}\rangle$ a deterministic automaton recognizing $L_{a,q}$. We construct the expected NFTA this way :

$$\mathcal{A}' = \langle Q', \mathcal{F}^{\Sigma}_{ext}, \Delta', F'\rangle$$

where :

- $Q' = \bigcup_{(a,q)} P_{a,q}$
- $F' = \bigcup_{(a,q) | q \in F} F_{a,q}$
- $\Delta' =$
  - $\star$ $a \longrightarrow p^0_{a,q}$ for all (a,q)
  - $\star$ $@(p,p') \longrightarrow p''$ if $p, p'' \in P_{b,q}$, $p' \in F_{a,q'}$ with $\delta_{b,q}(p,q') = p''$ for some b, q, a, q'

$\Leftarrow$) Let $\mathcal{A} = \langle Q, \mathcal{F}^{\Sigma}_{ext}, F, \Delta\rangle$ be a NFTA recognizing $ext(L)$. We construct the expected NFHA this way :

$$\mathcal{A}' = \langle Q, \Sigma, F, \Delta'\rangle$$

where for all (a,q), $a(R_{a,q}) \longrightarrow q \in \Delta'$ where $R_{a,q}$ is the language recognized by the automaton :

$$B_{a,q} = \langle Q, Q, I_{a,q}, F_{a,q}, \Delta_{a,q}\rangle$$

with :

- $I_{a,q} = \{p \in Q \mid a \longrightarrow p \in \Delta\}$
- $F_{a,q} =$
  - $\star$ $\{q\}$ if $q \in F$ or if there exists q', q'' such that $@(q',q) \longrightarrow q'' \in \Delta$
  - $\star$ $\varnothing$ else
- $\Delta_{a,q} = \{(q_1, q_2, q_3) \mid @(q_1,q_2) \longrightarrow q_3 \in \Delta\}$

Exercise 3 : Complexity

Show that the emptiness problem for NFHA(NFA) is in PTIME.

Solution:

We run the emptiness algorithm used to dertermine the emptiness of a NFTA on the extension encoding of the automaton.

Exercise 4 : SUCH AWA

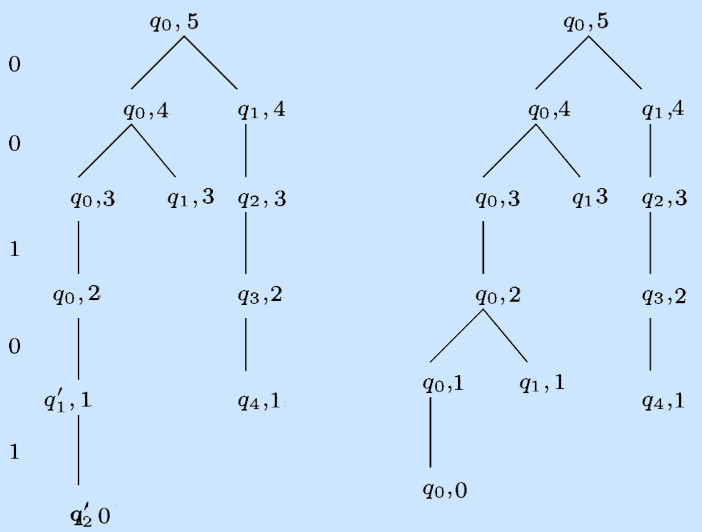
Definition 2 *If $\mathcal{X}$ is a set of propositional variables, let $\mathbb{B}(\mathcal{X})$ be the set of positive propositional formulae on $\mathcal{X}$, i.e., formulae generated by the grammar $\phi ::= \bot \mid \top \mid \phi \vee \phi \mid \phi \wedge \phi$.*

Definition 3 *A AWA (Alternating Word Automata) is a tuple $\mathcal{A} = (Q, \Sigma, Q_0, Q_f, \delta)$ where Σ is a finite set (alphabet), Q is a finite set (of states), $Q_0 \subseteq Q$ (initial states), $Q_f \subseteq Q$ (final states) and δ is a function from $Q \times \Sigma$ to $\mathbb{B}(Q)$ (transition function). A run of $\mathcal{A} = (Q, \Sigma, Q_0, Q_f, \delta)$ on a word w is a tree t labelled by $Q \times \mathbb{N}$ such that :*
— *if $w = \varepsilon$, then $t = (q_0, 0)$ with $q_0 \in Q_0$.*
— *if $w = a.w'$, then $t = (q_0, k)(t_1, \ldots, t_n)$ where k is the length of w, $q_0 \in Q_0$ and such that for all i, t_i is a run of w' on $(Q, \Sigma, \{q_i\}, Q_f, \delta)$ for some q_i satisfying $\{q_1, \ldots, q_n\} \models \delta(q_0, a)$.*

Definition 4 *We say that a run is accepting if every leaf of the form $(q, 0)$ satisfies that $q \in Q_f$. Notice that a run may have leaves of the form (q, i) with $i > 0$ if $\varnothing \models \delta(q_0, a)$. Those leaves are considered as 'success' leaves in this semantic. The language of a AWA is the set of words which have an accepting run.*

1. Let $\Sigma = \{0, 1\}$ and $\mathcal{A} = (Q, \Sigma, Q_0, Q_f, \delta)$ the AWA such that $Q = \{q_0, q_1, q_2, q_3, q_4, q_1', q_2'\}$, $Q_f = \{q_0, q_1, q_2, q_3, q_4\}$ and :
$$\delta = \{ \begin{array}{ll} q_0 0 \longrightarrow (q_0 \wedge q_1) \vee q_1' & q_0 1 \longrightarrow q_0 \\ q_1 0 \longrightarrow q_2 & q_1 1 \longrightarrow \top \\ q_2 0 \longrightarrow q_3 & q_2 1 \longrightarrow q_3 \\ q_3 0 \longrightarrow q_4 & q_3 1 \longrightarrow q_4 \\ q_4 0 \longrightarrow \top & q_4 1 \longrightarrow \top \\ q_1' 0 \longrightarrow q_1' & q_1' 1 \longrightarrow q_2' \\ q_2' 0 \longrightarrow q_2' & q_2' 1 \longrightarrow q_1' \} \end{array}$$
Give an example of an accepting computation of $\mathcal{A}$ on $w = 00101$ and an example of a non accepting computation of $\mathcal{A}$ on w.
2. Prove that for all AWA, we can compute in exponential time a non-deterministic automaton which accepts the same language.

Solution:



1. The left is non accepting but the right is.
2. Given $\mathcal{A} = (Q, \Sigma, Q_0, Q_f, \delta)$ an AWA, we produce $\mathcal{A}' = (2^Q, \Sigma, 2^{Q_0}, 2^{Q_f}, \delta')$ with :

$$\delta'(S, a) = \{S' | S' \models \bigwedge_{s \in S} \delta(s, a)\}$$