Automates d'arbre

TD n°5 : Hedges and Alternation

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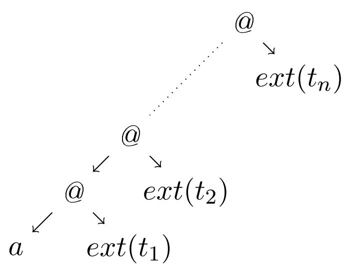
Exercise 1 : Extensions

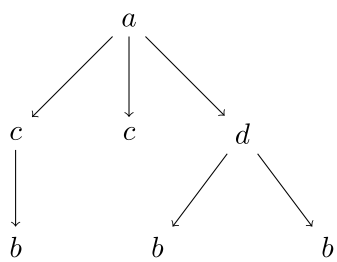
Definition 1 (extension encoding)
Let t be an unranked tree on Σ. Let $\mathcal{F}_{ext}^{\Sigma} = \{@(2)\} \cup \{a(0) \mid a \in \Sigma\}$. We define the ranked tree $ext(t)$ by induction on the size of t by :

- *for $a \in \Sigma$, $ext(a) = a$*
- *if $t = a(t_1, ..., t_n)$ with $n \geq 1$, $ext(t) = @(ext(a(t_1, ..., t_{n-1})), ext(t_n))$*

that is $ext(a(t_1, ..., t_n))$ is equal to :



Give the extension encoding of :



Exercise 2 : The soundess of the extension

Let L be a language of unranked trees. Prove that L is recognizable by a NFHA iff $ext(L)$ is recognizable by a NFTA.

Exercise 3 : Complexity

Show that the emptiness problem for NFHA(NFA) is in PTIME.

Exercise 4 : SUCH AWA

Definition 2 *If $\mathcal{X}$ is a set of propositional variables, let $\mathbb{B}(\mathcal{X})$ be the set of positive propositional formulae on $\mathcal{X}$, i.e., formulae generated by the grammar $\phi ::= \bot \mid \top \mid \phi \vee \phi \mid \phi \wedge \phi$.*

Definition 3 *A AWA (Alternating Word Automata) is a tuple $\mathcal{A} = (Q, \Sigma, Q_0, Q_f, \delta)$ where Σ is a finite set (alphabet), Q is a finite set (of states), $Q_0 \subseteq Q$ (initial states), $Q_f \subseteq Q$ (final states) and δ is a function from $Q \times \Sigma$ to $\mathbb{B}(Q)$ (transition function). A run of $\mathcal{A} = (Q, \Sigma, Q_0, Q_f, \delta)$ on a word w is a tree t labelled by $Q \times \mathbb{N}$ such that :*
- *if $w = \varepsilon$, then $t = (q_0, 0)$ with $q_0 \in Q_0$.*
- *if $w = a.w'$, then $t = (q_0, k)(t_1, \ldots, t_n)$ where k is the length of w, $q_0 \in Q_0$ and such that for all i, t_i is a run of w' on $(Q, \Sigma, \{q_i\}, Q_f, \delta)$ for some q_i satisfying $\{q_1, \ldots, q_n\} \models \delta(q_0, a)$.*

Definition 4 *We say that a run is accepting if every leaf of the form $(q, 0)$ satisfies that $q \in Q_f$. Notice that a run may have leaves of the form (q, i) with $i > 0$ if $\varnothing \models \delta(q_0, a)$. Those leaves are considered as 'success' leaves in this semantic. The language of a AWA is the set of words which have an accepting run.*

1. Let $\Sigma = \{0, 1\}$ and $\mathcal{A} = (Q, \Sigma, Q_0, Q_f, \delta)$ the AWA such that $Q = \{q_0, q_1, q_2, q_3, q_4, q_1', q_2'\}$, $Q_f = \{q_0, q_1, q_2, q_3, q_4\}$ and :
$$\begin{array}{lll} \delta = \{ & q_0 0 \longrightarrow (q_0 \wedge q_1) \vee q_1' & q_0 1 \longrightarrow q_0 \\ & q_1 0 \longrightarrow q_2 & q_1 1 \longrightarrow \top \\ & q_2 0 \longrightarrow q_3 & q_2 1 \longrightarrow q_3 \\ & q_3 0 \longrightarrow q_4 & q_3 1 \longrightarrow q_4 \\ & q_4 0 \longrightarrow \top & q_4 1 \longrightarrow \top \\ & q_1' 0 \longrightarrow q_1' & q_1' 1 \longrightarrow q_2' \\ & q_2' 0 \longrightarrow q_2' & q_2' 1 \longrightarrow q_1' \} \end{array}$$
Give an example of an accepting computation of $\mathcal{A}$ on $w = 00101$ and an example of a non accepting computation of $\mathcal{A}$ on w.
2. Prove that for all AWA, we can compute in exponential time a non-deterministic automaton which accepts the same language.