Automates d'arbre

TD n°4 : Logic and Hedges

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Exercise 1 : The power of Wsks

Produce formulae of WSkS for the following predicates :

- the set X has exactly two elements.
- the set X contains at least one string beginning with a 1.
- $x \leq_{lex} y$ where $\leq_{lex}$ is the lexicographic order on $\{1, ...k\}^*$.
- given a formula of WSkS ϕ with one free first-order variable, produce a formula of WSkS expressing that there is an infinity of words on $\{1, ..., k\}^*$ satisfying ϕ.

Solution:

- $$|X| \leq 2 \doteq \forall Y. Y \subseteq X \Rightarrow (Y = \varnothing \vee Sing(Y) \vee Y = X)$$
  $$|X| \geq 2 \doteq \exists x, y. x \neq y \wedge x \in X \wedge y \in X$$
  $$|X| = 2 \doteq |X| \leq 2 \wedge |X| \geq 2$$
- $$X \cap 1.\Sigma^* \neq \varnothing \doteq \exists x. x \in X \wedge 1 \leq x$$
- $$x \leq_{lex} y \doteq x \leq y \vee (\exists z. \bigvee_{i<j\leq k} (z.i \leq x \wedge z.j \leq y))$$
- $$X \models \phi \doteq \forall x, x \in X \Rightarrow \phi(x)$$
  $$\phi \text{ satisfied by an infinity of words } \doteq \forall X,\, X \models \phi \Rightarrow \exists Y,\, X \subsetneq Y \wedge Y \models \phi$$

Exercise 2 : The limit of Wsks

Prove that the predicate $x = 1y$ is not definable in WSkS.

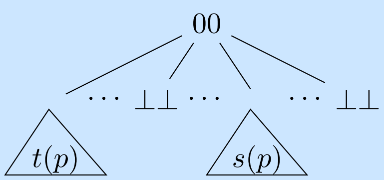
Solution:

We use the equivalence with recognizable tree languages. So we have to prove that $L = \{tra(x, y) \mid x = 1.y\}$ is not recognizable. Using the translation, we see that

$$L \cap \{t_i\sigma \mid t_i = 00(i\bot(x_1, ..., x_k), y_2, ..., y_k), i \in \{0, 1\}, \sigma \text{ closed substitution}\}$$

$$= \{tra(x, y) \mid x = 1.y \wedge y \in \{2, ..., k\}.\{1, ..., k\}^*\} = L'$$

So it is enough to prove that L' is not recognizable. Now elements of L' are of the form :



with $p \in \{2, ..., k\}.\{1, ..., k\}^*$, t and s injective and the height of t and s strictly increasing with p. You can reason by contradiction using the pumping lemma : for p large enough, using the pumping lemma, you can iterate a piece of $t(p)$ without touching $s(p)$ (or vice versa) while staying in L' which is absurd by injectivity.

Exercise 3 : Let's try to minimize

We consider the complete DFTA on $\mathcal{F} = \{f/2, g/2, a/0, b/0\}$ with states $\{q_a, q_b, q_f, q_g, \top, \bot\}$, finale state $\top$ and transitions :

— $a \longrightarrow q_a$
— $b \longrightarrow q_b$
— $f(q_a, q_b) \longrightarrow q_f$
— $f(q_f, q_b) \longrightarrow \top$
— $g(q_a, q_a) \longrightarrow q_q$
— $f(q_g, q_b) \longrightarrow \top$
— $h(q, q') \longrightarrow \top$ if $h \in \{f, g\}$, and $q = \top$ or $q' = \top$.
— $h(q, q') \longrightarrow \bot$ in all other cases.

Give the corresponding minimized algorithm obtained through the partition refinement algorithm.

Solution:

The initial partitioning is $P = \{(\top), (q_a, q_b, q_g, q_f, \bot)\}$.

Then, we can for instance distinguish q_a with q_f as $f(q_f, q_b) \longrightarrow \top$ but $Tf(q_a, q_b) \longrightarrow \bot$ and we do not have $(\bot\ P\ \top)$. This argument is also valid for (q_b, q_f),(q_b, q_g), (q_a, q_b),(q_a, q_g), and $(\bot_{,)}$. However, we have that $f(q_f, q_b) \longrightarrow \top$ and $Tf(q_g, q_b) \longrightarrow \top$, so we have $(q_f P' q_g)$. Finally, at the end of the first loop, $P' = \{(\top), (q_b), (q_a), (\bot), (q_f, q_g)\}$. If we try once more, it is stable, so we have the minimal automaton by merging q_f and q_g.

Exercise 4 : To the infinity...

Let $\Sigma = \{a, b\}$. Define a DFHA $\mathcal{A}$ such that $L(\mathcal{A})$ is the set of all trees such that "for every leaf labeled with a, there is an ancestor from which there is a path whose nodes are labeled with b". Here "ancestor" means strict ancestor and "from which there is a path" means that there is a path from a son of this ancestor to a leaf.

Solution:

$Q = \{q_a, q_b, q_\top\}$, $F = \{q_b, q_\top\}$ and $\Delta =$

- ⋆ $a(\epsilon) \longrightarrow q_a$
- ⋆ $a(q_\top^+) \longrightarrow q_\top$
- ⋆ $a((q_\top + q_a)^* q_a (q_\top + q_a)^*) \longrightarrow q_a$
- ⋆ $b(q_\top^+) \longrightarrow q_\top$
- ⋆ $b(\epsilon) \longrightarrow q_b$
- ⋆ $a(Q^* q_b Q^*) \longrightarrow q_\top$
- ⋆ $b(Q^* q_b Q^*) \longrightarrow q_b$
- ⋆ $b((q_\top + q_a)^* q_a (q_\top + q_a)^*) \longrightarrow q_a$