Automates d'arbre

TD n°4 : Logic and Hedges

Charlie Jacomme

October 03, 2017

Exercise 1 : The power of Wsks

Produce formulae of WSkS for the following predicates :

- the set X has exactly two elements.
- the set X contains at least one string beginning with a 1.
- $x \leq_{lex} y$ where $\leq_{lex}$ is the lexicographic order on $\{1, ...k\}^*$.
- given a formula of WSkS ϕ with one free first-order variable, produce a formula of WSkS expressing that there is an infinity of words on $\{1, ..., k\}^*$ satisfying ϕ.

Exercise 2 : The limit of Wsks

Prove that the predicate $x = 1y$ is not definable in WSkS.

Exercise 3 : Let's try to minimize

We consider the complete DFTA on $\mathcal{F} = \{f/2, g/2, a/0, b/0\}$ with states $\{q_a, q_b, q_f, q_g, \top, \bot\}$, finale state $\top$ and transitions :

— $a \longrightarrow q_a$

— $b \longrightarrow q_b$

— $f(q_a, q_b) \longrightarrow q_f$

— $f(q_f, q_b) \longrightarrow \top$

— $g(q_a, q_a) \longrightarrow q_q$

— $f(q_g, q_b) \longrightarrow \top$

— $h(q, q') \longrightarrow \top$ if $h \in \{f, g\}$, and $q = \top$ or $q' = \top$.

— $h(q, q') \longrightarrow \bot$ in all other cases.

Give the corresponding minimized algorithm obtained through the partition refinement algorithm.

Exercise 4 : To the infinity...

Let $\Sigma = \{a, b\}$. Define a DFHA $\mathcal{A}$ such that $L(\mathcal{A})$ is the set of all trees such that "for every leaf labeled with a, there is an ancestor from which there is a path whose nodes are labeled with b". Here "ancestor" means strict ancestor and "from which there is a path" means that there is a path from a son of this ancestor to a leaf.