Advanced Complexity

TD n°7

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1 The homework is back !

Exercise 1 : A many to one reduction
Assuming that BestClique is NP-hard, show that BestClique is ∇NP-complete.

Exercise 2 : Another many to one
Assuming NP $\neq$ coNP, show that BestClique $\notin$ NP.

Exercise 3 : Not a many to one
Assuming NP $\neq$ coNP, show that BestClique $\notin$ coNP.

2 The truth about the homework

Definition 1 *A language L is in* DP *if and only if there are two language $L_1 \in$ NP and $L_2 \in$ coNP such that $L = L_1 \cap L_2$*

The class DP (the D stands for differential) was introduced by Papadimitriou and Yannakis in [2]. Some problems were then proved to be complete for this class in [3], and the strictness properties where studied in [1].

Definition 2 *The* TSP *problem is defined as :*
— *INPUT : A distance matrix and an integer B.*
— *QUESTION : Is there a tour of length B or less ?*
The ExactTSP *problem is defined as :*
— *INPUT : A distance matrix and an integer B.*
— *QUESTION : Is the length of the shortest tour B ?*

Exercise 4 : Money Money Money

1. Show that TSP is NP-complete.
   Hint : HAMILTONPATH *is* NP*-complete*
2. Why it is unlikely that ExactTSP $\in$ NP or ExactTSP $\in$ coNP ?
3. Prove that ExactTSP $\in$ DP.
4. Modify the reduction from 3SAT to HAMILTONPATH, so that , wheter or not the expressions are satisfiable, the produced graph will always contain a broken hamiltonian path, that is, two node-disjoint paths that cover all nodes.
   You do not need to know the reduction of 3SAT *to* HAMILTONPATH*, you just need to know that it produces a graph with a "starting" node and and an "ending" node, and if there is only one not satisfied clause, then there is a broken hamiltonian path in the graph.*
5. Prove that ExactTSP is DP-complete.

Références

[1] E.W. Leggett and Daniel J. Moore. Optimization problems and the polynomial hierarchy. *Theoretical Computer Science*, 15(3) :279 – 289, 1981.

[2] C.H. Papadimitriou and M. Yannakakis. The complexity of facets (and some facets of complexity). *Journal of Computer and System Sciences*, 28(2) :244 – 259, 1984.

[3] Christos H. Papadimitriou and David Wolfe. The complexity of facets resolved. *Journal of Computer and System Sciences*, 37(1) :2 – 13, 1988.