1. **Exercise 1: Language theory**
   Show that the following problems are PSPACE-complete:
   
   1. **NFA Universality**
      - **INPUT**: a non-deterministic automaton $A$ over alphabet $\Sigma$
      - **QUESTION**: $L(A) = \Sigma^*$?
      - **Bonus**: what is the complexity of this problem for a DFA?
   
   2. **NFA Equivalence**
      - **INPUT**: two non-deterministic automata $A_1$ and $A_2$ over the same alphabet $\Sigma$
      - **QUESTION**: $L(A_1) = L(A_2)$
      - **Bonus**: what is the complexity of this problem for a DFA?
   
   3. **DFA Intersection Vacuity**
      - **INPUT**: deterministic automata $A_1, \ldots, A_m$ for some $m$
      - **QUESTION**: $\bigcap_{i=1}^m L(A_i) = \emptyset$?

   **Exercise 2: Did you get padding?**
   Show that if $P = \text{PSPACE}$, then $\text{EXPTIME} = \text{EXPSPACE}$.

   **Exercise 3: Too fast!**
   Show that $\text{ATIME}(\log n) \neq L$.

   **Exercise 4: Direct application**
   Show that $\text{EXPSPACE} = \text{AEXPTIME}$.
   **Hint**: You may use that if $f$ is space-constructible, then:
   
   $$\text{SPACE}(\text{poly}(f(n))) = \text{ATIME}(\text{poly}(f(n)))$$

   **Exercise 5: Closure under morphisms**
   Given a finite alphabet $\Sigma$, a function $f : \Sigma^* \rightarrow \Sigma^*$ is a morphism if $f(\Sigma) \subseteq \Sigma$ and for all $a = a_1 \cdots a_n \in \Sigma^*$, $f(a) = f(a_1) \cdots f(a_n)$ ($f$ is uniquely determined by the value it takes on $\Sigma$).
   
   1. Show that $\text{NP}$ is closed under morphisms, that is: for any language $L \in \text{NP}$, and any morphism $f$ on the alphabet of $L$, $f(L) \in \text{NP}$.
   
   2. Show that if $P$ is closed under morphisms, then $P = \text{NP}$.