Exercise 1: Warm up

Show that the following problems are NL-complete:

1. Deciding if a non-deterministic automaton $\mathcal{A}$ accepts a word $w$.
2. Deciding if a directed graph is strongly connected is NL-complete.
3. Deciding if a directed graph has a cycle.

Exercise 2: Restrictions of the SAT problem

1. Let 3-SAT be the restriction of SAT to clauses consisting of at most three literals (called 3-clauses). In other words, the input is a finite set $S$ of 3-clauses, and the question is whether $S$ is satisfiable. Show that 3-SAT is NP-complete for logspace reductions (assuming SAT is).
2. Let 2-SAT be the restriction of SAT to clauses consisting of at most two literals (called 2-clauses). Show that 2-SAT is in P, using proofs by resolution.
3. Show that 2-UNSAT (i.e., the unsatisfiability of a set of 2-clauses) is NL-complete.
4. Conclude that 2-SAT is NL-complete.

Exercise 3: Space hierarchy theorem

Using a diagonal argument, prove that for two space-constructible functions $f$ and $g$ such that $f(n) = o(g(n))$ (and as always $f, g \geq \log$) we have $\text{SPACE}(f(n)) \subseteq \text{SPACE}(g(n))$. 