Advanced Complexity

TD n°2 : SPACE and NL

Charlie Jacomme

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Exercise 1 : Warm up

Show that the following problems are NL-complete :

1. Deciding if a non-deterministic automaton $\mathcal{A}$ accepts a word w.
2. Deciding if a directed graph is strongly connected is NL-complete.
3. Deciding if a directed graph has a cycle.

Exercise 2 : Restrictions of the SAT problem

1. Let 3-SAT be the restriction of SAT to clauses consisting of at most three literals (called 3-clauses). In other words, the input is a finite set S of 3-clauses, and the question is whether S is satisfiable. Show that 3-SAT is NP-complete for logspace reductions (assuming SAT is).
2. Let 2-SAT be the restriction of SAT to clauses consisting of at most two literals (called 2-clauses). Show that 2-SAT is in P, using proofs by resolution.
3. Show that 2-UNSAT (i.e, the unsatisfiability of a set of 2-clauses) is NL-complete.
4. Conclude that 2-SAT is NL-complete.

Exercise 3 : Space hierarchy theorem

Using a diagonal argument, prove that for two space-constructible functions f and g such that $f(n) = o(g(n))$ (and as always $f, g \geq log$) we have $\mathsf{SPACE}(f(n)) \subsetneq \mathsf{SPACE}(g(n))$.