Probabilistic Aspects of Computer Science: Exercise 4

Exercise 1 *Given a PFA* $\mathcal{A} = (\mathcal{Q}, q_s, \mathcal{Q}_f, \delta)$ *on* Σ *and a word* $u \in \Sigma^\star$, *show that*

$$\mu^{acc}_{\mathcal{A},u} = \delta_u(q_s, \mathcal{Q}_f).$$

Exercise 2 *For any PFA* A *and a cut-point* $x \in [0,1]$, *there is a PFA* $\mathcal{B}$ *and a* $y \in [0,1]$ *such that* $\mathcal{L}_{<x}(\mathcal{A}) = \mathcal{L}_{>y}(\mathcal{B})$ *and* $\mathcal{L}_{\leqslant x}(A) = \mathcal{L}_{\geqslant y}(\mathcal{B})$.

Exercise 3 *Let* $\Sigma = \{0,1\}$. *Consider the PFA* $\mathcal{A} = (Q, q_s, Q_f, \delta)$ *where* $\mathcal{Q} = (q_s, q_a, q_r)$, $\mathcal{Q}_f = q_a$ *and* δ *is defined as follows:*

- $\delta(q_s, 0, q_s) = \delta(q_s, 0, q_r) = \frac{1}{2}$
- $\delta(q_s, 1, q_s) = \delta(q_s, 1, q_a) = \frac{1}{2}$
- $\delta(q_a, 0, q_a) = \delta(q_a, 1, q_a) = \frac{1}{2}$
- $\delta(q_r, 0, q_r) = \delta(q_r, 1, q_r) = \frac{1}{2}$

Compute $\mathcal{L}_{>1/2}(\mathcal{A})$. *Is it regular?*

Exercise 4 *Let* $\mathcal{A} = (\mathcal{Q}, q_s, \mathcal{Q}_f, \delta)$ *be a PFA defined over four states* $\mathcal{Q} = \{1,2,3,4\}$ *and on alphabet* $\Sigma = \{a, b\}$. *Let* $q_s = \{1\}$ *and* $Q_f = \{3\}$ *be the initial and final steps respectively. Transitions are defined by the following matrices:*

$$\begin{pmatrix} 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \textit{and} \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Compute the dsitribution $\mu^{acc}_{\mathcal{A},ab}$