Exercise 1 Given a PFA $A = (Q, q_s, Q_f, \delta)$ on $\Sigma$ and a word $u \in \Sigma^*$, show that

$$\mu^\text{acc}_{A,u} = \delta_u(q_s, Q_f).$$

Exercise 2 For any PFA $A$ and a cut-point $x \in [0, 1]$, there is a PFA $B$ and a $y \in [0, 1]$ such that $L_{\leq x}(A) = L_{> y}(B)$ and $L_{\leq x}(A) = L_{\geq y}(B)$.

Exercise 3 Let $\Sigma = \{0, 1\}$. Consider the PFA $A = (Q, q_s, Q_f, \delta)$ where $Q = (q_s, q_a, q_r)$, $Q_f = q_a$ and $\delta$ is defined as follows:

$$\begin{align*}
\bullet \delta(q_s, 0, q_s) = \delta(q_s, 0, q_r) &= \frac{1}{2} \\
\bullet \delta(q_a, 0, q_a) = \delta(q_a, 1, q_a) &= \frac{1}{2} \\
\bullet \delta(q_r, 0, q_r) = \delta(q_r, 1, q_r) &= \frac{1}{2}
\end{align*}$$

Compute $L_{> \frac{1}{2}}(A)$. Is it regular?

Exercise 4 Let $A = (Q, q_s, Q_f, \delta)$ be a PFA defined over four states $Q = \{1, 2, 3, 4\}$ and on alphabet $\Sigma = \{a, b\}$. Let $q_s = \{1\}$ and $Q_f = \{3\}$ be the initial and final steps respectively. Transitions are defined by the following matrices:

$$\begin{pmatrix}
0 & \frac{3}{4} & \frac{1}{4} & 0 \\
0 & 1 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & \frac{1}{2} & \frac{1}{2} & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

Compute the distribution $\mu^\text{acc}_{A,ab}$.