Partial Order Reduction for Security Protocols

68NQRT Seminar

Lucca Hirschi, David Baelde and Stéphanie Delaune

15th December, 2016


Google
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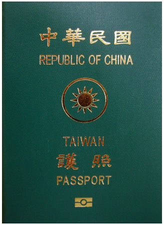
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REPUBLIC OF CHINA
TAIWAN
護照
PASSPORT


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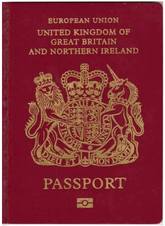
VOTE

wins (BlackHat'15)

wins (FMSE'08)



wins (CSF'10)

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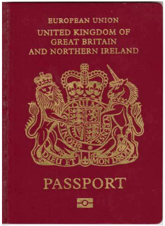

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concurrent programs + unsecure network + active attacker

⇝ tricky attacks, hard to detect/avoid

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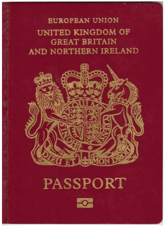

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concurrent programs + unsecure network + active attacker

⇝ tricky attacks, hard to detect/avoid

⇝ need a mathematical framework to analyze protocols: **formal methods**

Symbolic Model

Symbolic attacker ( ) controls all the network:

Symbolic Model

[$\{n\}_k$: symmetric encryption]

Symbolic attacker () controls all the network:

- eavesdrops messages

$$\text{Alice} \xrightarrow{\{n\}_k} \text{Bob}$$

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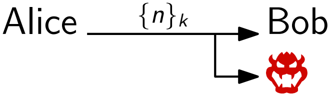

Symbolic Model

Symbolic attacker () controls all the network:

- eavesdrops messages
- builds new messages, applies crypto primitives

$\Big($ knows $\{n\}_k$ and $k\Big) \Rightarrow$
$\Big($ knows $n\Big)$

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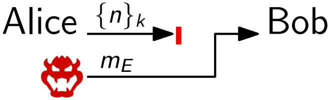

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But 👹 cannot break crypto primitives.

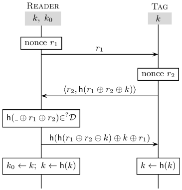
Symbolic model, pros & cons:

⊖ less precise than computational model (*i.e.,* no assumption on primitives)

⊕ allows for automation

Dolev, Yao: On the Security of Public Key Protocols. FOCS'81

Introduction: Formal Methods for Security Protocols



Protocol's specification

Security goal
(*e.g.*, Secrecy)

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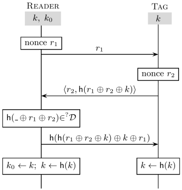
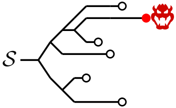



```
(* ---------------------------------------------------- *)
(*              ORIGINAL LAK                            *)
(* ---------------------------------------------------- *)

let LAK = ! new k:bitstring;
            out(t, enc(secretK, k));
          !    (        (*  TAG   *)
             in(t,x:bitstring);
             new r1:bitstring;
             let m = h(xor(x, r1, k)) in
             let mout = (r1,m) in
             event TOut(k,r1, mout);
             out(t, mout);

             in(t, y:bitstring);
             event TIn(k,r1,y);

             if y = h(xor(m,x,k)) then
             event TOk(k,r1,x);
             out(t, ok)
            ) | (   (* READER *)
             new r0:bitstring;
             out(r, r0);
             in(r, (x1:bitstring,x2:bitstring));
             event RIn(k,r0,(x1,x2));

             if x2 = h(xor(r0, x1, k)) then
             event ROk(k, r0, x1);

             event ROut(k,r0, h(xor(x2,r0,k)));
             out(t, h(xor(x2,r0,k)))
            ).

process LAK
```

Protocol's specification ⟶ Protocol's model
(symbolic model)

Reachability
in a model



Security goal ⟶ Modelization of the property
(*e.g.*, Secrecy)
Reachability(State$_{\text{bad}}$)

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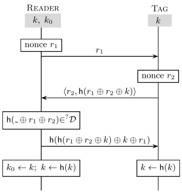


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process LAK
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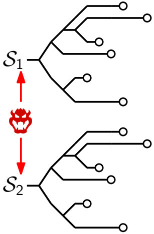
Protocol's specification → Protocol's model
(symbolic model)

Equivalence
of two models



Privacy goal → Modelization of the property
(*e.g.*, Unlinkability)

$!\nu\,\mathsf{id}\,!\nu\,\mathsf{Sess}.\ P \approx !\nu\,\mathsf{id}.\,\nu\,\mathsf{Sess}.\ P$

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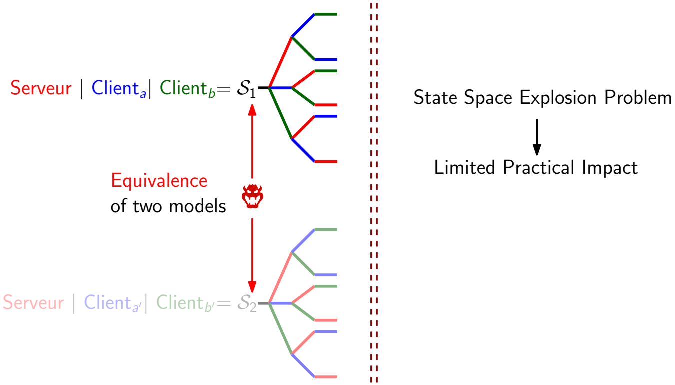

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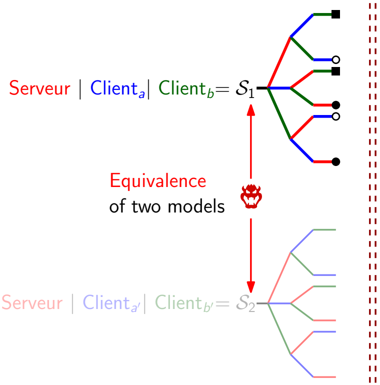

State Space Explosion Problem

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Limited Practical Impact

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Detect Redundant States

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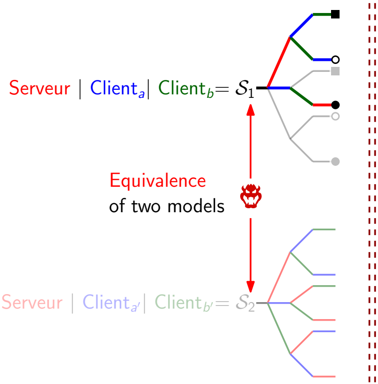

State Space Explosion Problem

Limited Practical Impact

Detect and Eliminate Redundant States

Problem

Issue: Limited practical impact

Too slow. – Bottleneck: state space explosion

e.g., verification of P.A.: 1 session $\rightarrow$ 1 sec. vs. 2 sessions $\rightarrow$ 9 days

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Our Contribution

Partial Order Reduction techniques:

- adequate with respect to specificities of security setting
- work for reachability and trace equivalence
- very effective in practice (implem $+$ bench)

Outline

I Model

II Big Picture

III Compression

IV Reduction

V Applications

VI Conclusion

I : Model

Applied-π - Term Algebra

Model of messages:

- Exchanged messages $=$ terms
- Crypto. primitives $=$ algebraic relations

Applied-π - Term Algebra

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Terms Algebra: signature $+$ equational theory.

Example: symmetric encryption

- symbols: $\mathsf{enc}(\circ, \circ),\ \mathsf{dec}(\circ, \circ)$
- equation: $\mathsf{dec}(\mathsf{enc}(x, y), y) =_{\mathsf{E}} x$

Applied-π - Syntax

Protocols $\rightsquigarrow$ process calculus (*i.e.,* applied pi calculus)

- Process: P, Q

P, Q	$:=$	0	null
	$\mid$	$\mathrm{in}(c, x).P$	input
	$\mid$	$\mathrm{out}(c, m).P$	output
	$\mid$	$\texttt{if}\ u = v\ \texttt{then}\ P\ \texttt{else}\ Q$	conditional
	$\mid$	$P \mid Q$	parallel
	$\mid$	$!\,\nu\vec{n}.P$	replication

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- **Frame** (ϕ): the set of messages revelead to
  $\leadsto$ intuition: 's knowledge

$$\phi = \{ \underbrace{w_1}_{\text{handle}} \mapsto \underbrace{\mathrm{enc}(m, k)}_{\text{out. message}}; w_2 \mapsto k \}$$

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- **Configuration:** $A = (\mathcal{P}; \phi)$

Applied-π - Semantics

- Recipes: are terms built using handles

$$e.g.,\quad \begin{array}{l} R = \mathrm{dec}(w_1, w_2) \\ R\phi =_{\mathsf{E}} m \end{array} \qquad \text{for } \phi = \{w_1 \mapsto \mathrm{enc}(m,k), w_2 \mapsto k\}$$

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- Semantics of configurations:
  - Protocol's output:

  $$(\{\mathrm{out}(c,u).P\} \cup \mathcal{P}; \phi) \xrightarrow{\mathrm{out}(c,w)} (\{P\} \cup \mathcal{P}; \phi \cup \{w \mapsto u\}) \quad \text{if } w \text{ fresh}$$

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  - $+$ expected rules for conditional and other constructs

$\rightsquigarrow$ controls all the network

Applied-π - Trace Equivalence

1. Reachability (*e.g.,* secret, authentification) and
2. Trace equivalence (*e.g.,* anonymity, unlinkability).

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Trace Equivalence

$A \sqsubseteq B$: for any $A \xrightarrow{\mathrm{tr}} A'$ there exists $B \xrightarrow{\mathrm{tr}} B'$ such that $\Phi(A') \sim \Phi(B')$.

$$A \approx B, \text{ when } A \sqsubseteq B \text{ and } B \sqsubseteq A.$$

(bisimulation: too strong)

II : Big Picture

Redundancies

- Motivation: Improve algorithms checking trace equivalence
- How: Remove redundant interleavings via a reduced semantics

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Two types of redundancies:

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- what about trace equivalence ($\approx$) ?

$$\text{e.g., } (\mathsf{in}(c_1, x) \mid \mathsf{out}(c_2, m)) \not\approx (\mathsf{out}(c_2, m).\mathsf{in}(c_1, x))$$

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- $\rightsquigarrow$ same swaps are possible ($\equiv$ same sequential dependencies)

Big Picture

$$
\boxed{\begin{array}{c} \longrightarrow \\ \approx \end{array}}
\xRightarrow[\text{Strong Sym: } \approx = \approx_a]{\text{Annot. Sem.}}
\boxed{\begin{array}{c} \longrightarrow_a \\ \approx_a \end{array}}
\xRightarrow[\text{Theorem 1: } \approx_a = \approx_c]{\text{Compression}}
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\xRightarrow[\text{Theorem 2: } \approx_c = \approx_r]{\text{Reduction}}
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$$

Required properties

$\rightarrow_r$ is such that:

- reachability properties coincide on $\rightarrow_r$ and $\rightarrow$;
- for **action-determinate** processes, trace-equivalence coincides on $\rightarrow_r$ and $\rightarrow$.

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A is *action-deterministic* if: two actions in parallel must be $\neq$

Attacker knows to/from whom he is sending/receiving messages.

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D. Baelde, S. Delaune and L. Hirschi: Partial Order Reduction for Security Protocols. CONCUR'15

D. Baelde, S. Delaune and L. Hirschi: A reduced semantics for deciding trace equivalence using constraint systems. POST'14

Outline

Annotated Semantics

- embeds labels into produced actions
- one can extract sequential dependencies from labelled actions

e.g., $\mathrm{in}(c_1, x) \mid \mathrm{out}(c_2, m) \xrightarrow{[\mathrm{out}(c_2,w)]^{1.2}.[\mathrm{in}(c_1,M_1)]^{1.1}}_a \cdot$ labels: in parallel

while $\mathrm{out}(c_2, m).\mathrm{in}(c_1, x) \xrightarrow{[\mathrm{out}(c_2,w)]^{1}.[\mathrm{in}(c_1,M_1)]^{1}}_a \cdot$ labels: in sequence

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Strong Symmetry Lemma

- mismatch on labels $\rightsquigarrow$ systematically used to show $\not\approx$
- for action-deterministic, ($\approx +$ labels) coincides with $\approx$

Compression - Intuitions

The Idea

Follow a particular **strategy** that reduces the number of choices by looking at the **nature** of available actions.

Polarities of processes:

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(Replication: $!\, \nu \overrightarrow{n}.\ P$ is *positive* but releases the focus)

Compression - Example

$$\mathcal{P} = \{ \quad !\ \nu n.\ \mathsf{in}(c, x).\mathsf{out}(c, \mathsf{enc}(\langle x, n \rangle\}, k)).0 \quad \}$$

Compressed interleavings:
$t =$

Compression - Example

$$\mathcal{P} = \{ \quad !\nu n.\ \mathrm{in}(c, x).\mathrm{out}(c, \{< x, n >\}_k).0; \\ \boxed{\mathrm{in}(\mathrm{c}_1, x).\mathrm{out}(c_1, \mathrm{enc}(\langle x, n_1\rangle, k)).0} \quad \}$$

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$t = \mathrm{sess}(a, c_1)$

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Compressed interleavings:
$t =\ \mathsf{sess}(a,c_1).\mathsf{in}(c_1,X_1).\mathsf{out}(c_1,w_1)$

Only traces of the form:
$\mathsf{sess}_1.\mathsf{in}_1.\mathsf{out}_1.\ \mathsf{sess}_2.\mathsf{in}_2.\mathsf{out}_2.\ \ldots$

Compression - Results

Reachability:

- Soundness: $A \xrightarrow{t}_c A' \;\Rightarrow\; A \xrightarrow{t} A'$
- Completeness: for complete execution $A \xrightarrow{t} A' \;\Rightarrow$ $\exists t_c,$ permutation of $t,\; A \xrightarrow{t_c}_c A'$

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Equivalence:

Theorem: $\approx_c = \approx$

Let A and B be two action-deterministic configurations.

$$A \approx B \text{ if, and, only if, } A \approx_c B.$$

Outline

Reduction - Intuitions

By building upon $\rightarrow_c, \approx_c$:

- compressed semantics produces *blocks* of actions of the form:

$$b = (\mathit{sess}).\mathit{in} \ldots \mathit{in}.\mathit{out} \ldots \mathit{out}$$

- but we still need to make choices (which *positive* process/block?)
- some of them are redundant.

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- some of them are redundant.

$$P = \text{in}(c_1, x).\text{out}(c_1, m_1) \mid \text{in}(c_2, y).\text{out}(c_2, m_2)$$

Compressed traces:

- $\text{tr}_1 = \text{in}(c_1, M_1).\text{out}(c_1, w_1).\text{in}(c_2, M_2).\text{out}(c_2, w_2)$
- $\cancel{\text{tr}_2 = \text{in}(c_2, M_2).\text{out}(c_2, w_2).\text{in}(c_1, M_1).\text{out}(c_1, w_1)}$
  when M_1 does not use w_2

Reduction - Monoid of traces

Definition

Given a frame Φ, the relation $\equiv_\Phi$ is the smallest equivalence over compressed traces such that:

- $t.b_1.b_2.t' \equiv_\Phi t.b_2.b_1.t'$ when $b_1 \parallel b_2$, and
- $t.b_1.t' \equiv_\Phi t.b_2.t'$ when $(b_1 =_E b_2)\Phi$.

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- $t.b_1.t' \equiv_\Phi t.b_2.t'$ when $(b_1 =_\mathsf{E} b_2)\Phi$.

Lemma

If $A \xrightarrow{t}_c A'$. Then $A \xrightarrow{t'}_c A'$ for any $t' \equiv_{\Phi(A')} t$.

Goal: explore one trace per equivalence class.

Reduced semantics

We assume an arbitrary order $\prec$ over blocks priority order.

Semantics (informal)

$$\frac{A \xrightarrow{t}_r A' \quad A' \xrightarrow{b}_c A''}{A \xrightarrow{t.b}_r A'} \quad \text{if } t \ltimes b$$

Informally, $t \ltimes b$ means:

> *there is no way to swap b towards the beginning of t before a block $b_0 \succ b$ (even by modifying recipes)*

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t is Φ-minimal if there is no $t' \equiv_\Phi t$ such that $t' \prec_{\text{lex}} t$

If $A \xrightarrow{t}_c A'$ then t is $\Phi(A')$-minimal if, and only if, $A \xrightarrow{t}_r A'$.

Theorem

$\approx = \approx_r$ for action-deterministic configurations.

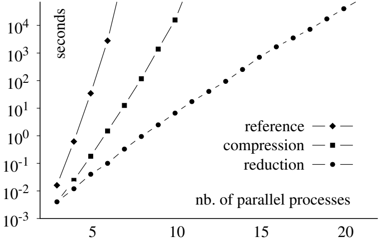
V : Applications

Benchmarks

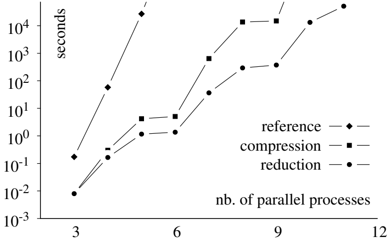
We implemented compression/reduction in APTE by adapting well established techniques based on:

- symbolic semantics (abstract inputs);
- constraint solving procedures.

tr$\ltimes$*b*: a new type of constraints



Toy example



Wide Mouthed Frog

All benchmarks & instructions for reproduction:
`www.lsv.ens-cachan.fr/~hirschi/apte_por`

VI : Conclusion

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- New optimizations: compression and reduction;
- applied to trace equivalence checking;
- implementation in APTE.

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Future Work

1. drop action-deterministic assumption
2. impact of the choice of $\prec$
3. POR for backward research
4. study others redundancies $\rightsquigarrow$ recognize symmetries ?

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Any question?

Compressed semantics - Definition

$\mathcal{P}$ is **initial** if $\forall P \in \mathcal{P}$, P is *positive*or replicated.

Semantics:

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Semantics:

$$\text{START/IN} \qquad \frac{\mathcal{P} \text{ is initial} \quad (P; \Phi) \xrightarrow{\mathrm{in}(c,M)} (P'; \Phi)}{(\mathcal{P} \uplus \{P\}; \varnothing; \Phi) \xrightarrow{\mathtt{foc}(\mathrm{in}(c,M))}_c (\mathcal{P}; P'; \Phi)}$$

$$\text{POS/IN} \qquad \frac{(P; \Phi) \xrightarrow{\mathrm{in}(c,M)} (P'; \Phi)}{(\mathcal{P}; P; \Phi) \xrightarrow{\mathrm{in}(c,M)}_c (\mathcal{P}; P'; \Phi)}$$

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POS/IN
$$\frac{(P;\Phi) \xrightarrow{\text{in}(c,M)} (P';\Phi)}{(\mathcal{P};P;\Phi) \xrightarrow{\text{in}(c,M)}_c (\mathcal{P};P';\Phi)}$$

RELEASE
$$\frac{P \text{ negative}}{(\mathcal{P};P;\Phi) \xrightarrow{\mathtt{rel}}_c (\mathcal{P} \uplus \{P\};\varnothing;\Phi)}$$

NEG/α
$$\frac{(\{P\};\Phi) \xrightarrow{\alpha} (\mathcal{P}';\Phi')}{(\mathcal{P} \uplus \{P\};\varnothing;\Phi) \xrightarrow{\alpha}_c (\mathcal{P} \uplus \mathcal{P}';\varnothing;\Phi')} \quad \alpha \in \{\mathtt{par}, \mathtt{zero}, \text{out}(\_,\_)\}$$

+ Repl/In

Reduced semantics

We assume an arbitrary order $\prec$ over blocks (without recipes/messages): priority order.

Semantics

$$\frac{}{A \xrightarrow{\epsilon}_r A}$$

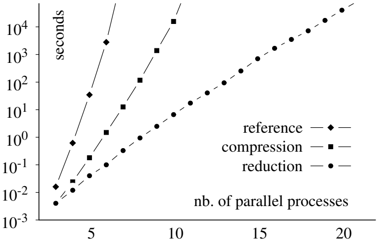
$$\frac{A \xrightarrow{\mathrm{tr}}_r (\mathcal{P}; \varnothing; \Phi) \quad (\mathcal{P}; \varnothing; \Phi) \xrightarrow{b}_c A'}{A \xrightarrow{\mathrm{tr}.b}_r A'} \quad \text{if } \mathrm{tr} \ltimes b' \text{ for all } b' \text{ with } (b' =_E b)\Phi$$

Availability

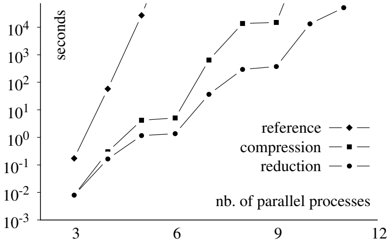
A block b is *available* after tr, denoted $\mathrm{tr} \ltimes b$, if:

- either $\mathrm{tr} = \epsilon$
- or $\mathrm{tr} = \mathrm{tr}_0.b_0$ with $\neg(b_0 \| b)$
- or $\mathrm{tr} = \mathrm{tr}_0.b_0$ with $b_0 \| b$, $b_0 \prec b$ and $\mathrm{tr}_0 \ltimes b$.

Benchmarks



Toy example (Π_i(in.out))



Wide Mouthed Frog

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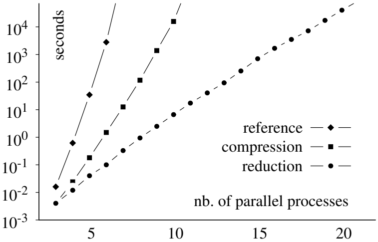
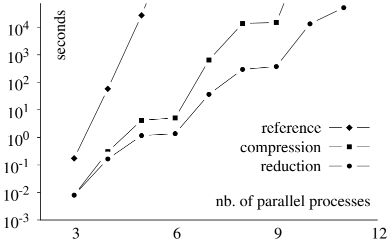


Toy example ($\Pi_i(in.out)$)



Wide Mouthed Frog

Maximum number of parallel processes verifiable in 20 hours:

Protocol	ref	comp	red
Yahalom (3-party)	4	5	5
Needham Schroeder (3-party)	4	6	7
Private Authentication (2-party)	4	7	7
E-Passport PA (2-party)	4	7	9
Denning-Sacco (3-party)	5	9	10
Wide Mouthed Frog (3-party)	6	12	13

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