A reduced semantics for deciding trace equivalence using constraint systems

CEA - Seminar

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joint work with	David Baelde LSV	*and*	Stéphanie Delaune LSV

ENS DE LYON

Introduction

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$\leadsto$ we need formal verification of crypto protocols

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- undecidable in general

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- decidable if we consider a bounded nb. of sessions

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Issue: Limited practical impact

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Our Contribution

Reduce search space of equivalence checking using POR ideas by eliminating a lot of redundancies (for simple processes).

David Baelde, Stéphanie Delaune, and Lucca Hirschi.
A reduced semantics for deciding trace equivalence using constraint systems.
In Martín Abadi and Steve Kremer, editors, *Proceedings of the 3rd International Conference on Principles of Security and Trust (POST'14)*, Lecture Notes in Computer Science, Grenoble, France, April 2014. Springer.
To appear.

Applied-π

Terms

$\mathcal{T}$: set of terms + equational theory. *e.g.,* $\mathrm{dec}(\mathrm{enc}(m, k), k) = m$.

Simple Processes

- $P_c ::= 0 \mid \mathtt{in}(c, x) \mid \mathtt{out}(c, m).P_c \mid \mathtt{if}\ T\ \mathtt{then}\ P_c\ \mathtt{else}\ P_c$
- $P_s ::= P_{c_1} | P_{c_2} | \ldots P_{c_n} \qquad c_i \neq c_j$

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Semantics

$$(\{\mathtt{out}(c,m).P\} \uplus \mathcal{P}; \Phi) \xrightarrow{\nu w.\mathtt{out}(c,w)} (\{P\} \uplus \mathcal{P}; \Phi \cup \{w \triangleright m\})$$
if $T \ \wedge\ w$ fresh in Φ

$$(\{\mathtt{in}(c,x).P\} \uplus \mathcal{P}; \Phi) \xrightarrow{\mathtt{in}(c,t)} (\{P[x \mapsto u]\} \cup \mathcal{P}; \Phi)$$
if $t\Phi = u \ \wedge\ fv(t) \subseteq \mathrm{dom}(\Phi)$

Example

Wide Mouth Frog

$$\begin{array}{rcl} \text{Alice} \rightarrow \text{Serveur} & : & \text{enc}(k', k_A) \\ \text{Serveur} \rightarrow \text{Bob} & : & \text{enc}(k', k_B) \\ \text{Alice} \rightarrow \text{Bob} & : & \text{enc}(m, k') \end{array}$$

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| in(s,x). if x = enc(y,ka) then out(s,enc(y,kb))
| in(b,x). if x = enc(y,kb) then
    b(z). if z = enc(w,y) then  ...
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$$\Phi = \varnothing$$

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- $P = (\mathcal{P}; \Phi)$;
- Φ: attacker's knowledge.

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Properties:

1. Reachability (secret, authentification) and
2. Equivalence (anonymity, unlikability).

Trace Equivalence

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- $\Phi \sim \Phi' \iff (\forall M, N,\ M\Phi = N\Phi \iff M\Phi' = N\Phi')$
- $A \approx B \iff \forall A \xrightarrow{s} A',\ \exists B',\ B \xrightarrow{s} B' \wedge \Phi_{A'} \sim \Phi_{B'}$ and conversely.

Trace equivalence allows to model anonymity, unlinkability, etc.

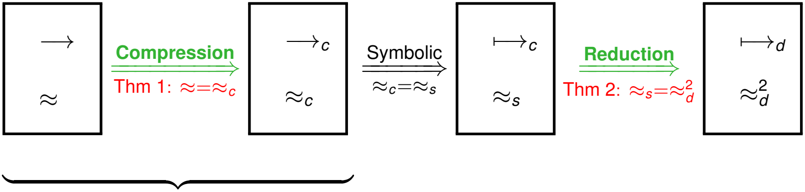
Big Picture

Goal

- Motivation: Improve algorithms checking trace equivalence for simple processes
- How: Dramatically decrease the number of interleavings to consider via a reduced semantics

$$\boxed{\begin{matrix}\longrightarrow \\ \approx\end{matrix}} \xRightarrow[\text{Thm 1: } \approx = \approx_c]{\textbf{Compression}} \boxed{\begin{matrix}\longrightarrow_c \\ \approx_c\end{matrix}} \xRightarrow[\approx_c = \approx_s]{\text{Symbolic}} \boxed{\begin{matrix}\longmapsto_c \\ \approx_s\end{matrix}} \xRightarrow[\text{Thm 2: } \approx_s = \approx_d^2]{\textbf{Reduction}} \boxed{\begin{matrix}\longmapsto_d \\ \approx_d^2\end{matrix}}$$

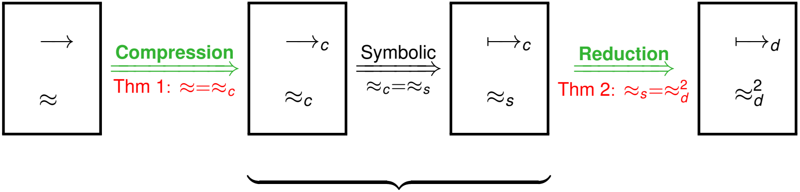
Big Picture



Grouping actions:

- generalization of the idea "force to perform output as soon as possible"
- $\rightarrow_c$ only explores specific traces
- Theorem 1: $\approx = \approx_c$

Big Picture



Symbolic semantics:

- classic step adapted for $\rightarrow_c$

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Analyze dependencies:

- force one order for independent (parallel) actions
- analyze dependencies "on the fly"
- $\longmapsto_d$ explores even less traces
- Theorem 2: $\approx_s=\approx_d^2$

Outline

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Grouping actions into *blocks*

$$\mathtt{in}(c,\_)\ldots\mathtt{in}(c,\_).\mathtt{out}(c,\_)\ldots\mathtt{out}(c,\_)$$

via a focused semantics $\rightarrow_c$.

Compression - Example

Basic rules of $\rightarrow_c$:

- choose a basic process $P_i \in \mathcal{P}$, it is now under focus;
- focus: only P_i can perform actions
- P_i can release the focus only if:
  - it has performed a block IO ($>$ 1 input, $>$ 1 output) and
  - it can not perform an output any more.

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Example

Consider $P = P_1 \mid P_2$ with $P_i = \mathtt{in}(c_i, x).\mathtt{in}(c_i, y).\mathtt{out}(c_i, \langle x, y \rangle)$.

- Semantics $\rightarrow_c$ explores only two interleavings of 6 actions:

  $\mathtt{in}(c_1, x_1).\mathtt{in}(c_1, y_1).\mathtt{out}(c_1, w_1).\mathtt{in}(c_2, x_2).\mathtt{in}(c_2, y_2).\mathtt{out}(c_2, w_2)$

  and

  $\mathtt{in}(c_2, x_2).\mathtt{in}(c_2, y_2).\mathtt{out}(c_2, w_2).\mathtt{in}(c_1, x_1).\mathtt{in}(c_1, y_1).\mathtt{out}(c_1, w_1)$

- semantics $\rightarrow$ explores 20 such interleavings.

Compression - Result

The semantics $\rightarrow_c$ induces a compressed trace equivalence $\approx_c$

Theorem 1

$$A \approx B \iff A \approx_c B$$

Key ideas

- symmetric: remove same interleavings on both sides
- completeness: in any execution, we can swap two actions on different channels (simple processes)

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Benefits

- first optimization that decreases (possibly exponentially many) interleavings to consider
- easy to implement
- allow us to reason with *macro-actions i.e.,* blocks
  $\rightsquigarrow$ reduced semantics

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Symbolic calculus - 1

Inputs messages: infinitely branching $\rightsquigarrow$ symbolic calculus.

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$$P = \mathtt{out}(c,k).\mathtt{in}(c,x).\mathtt{out}(c,\langle k,x\rangle).\mathtt{in}(c,y)$$

leads to

$$\mathcal{S} = \{\{w\} \vdash^?_X x, \{w,w'\} \vdash^?_Y y\}$$
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Symbolic process

$$(\mathcal{P}; \Phi; \mathcal{S})$$

Symbolic Calculus - 2

Semantics

$$(\{\mathtt{out}(c,m).P\} \uplus \mathcal{P}; \Phi; \mathcal{S}) \xrightarrow{\nu w.\mathtt{out}(c,X)} (\{P\} \uplus \mathcal{P}; \Phi \cup \{w \triangleright m\}; \mathcal{S})$$

if w fresh in ϕ

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Symbolic equivalence

$A \approx_s B \iff \forall A \overset{s}{\mapsto} A' \; \forall \Theta \in \mathcal{S}\mathrm{ol}(\Phi_{A'}, \mathcal{D}_{A'}),\ \exists B' \; B \overset{s}{\mapsto} B', \Theta \in \mathcal{S}\mathrm{ol}(\Phi_{B'}, \mathcal{D}_{B'})$ and $\Phi_{A'} \sim \Phi_{B'}$ and conversely.

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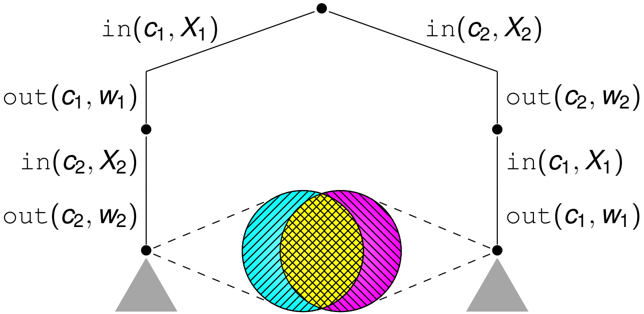
- There already exist several procedures checking equivalence between constraint systems
- Goal: starting with $\mapsto_c$ (compressed symbolic semantics), reduces the number of interleavings to explore

$$P = \texttt{in}(c_1, x_1).\texttt{out}(c_1, k_1).P_1 \mid \texttt{in}(c_2, x_2).\texttt{out}(c_2, k_2).P_2$$



Sebastian Mödersheim, Luca Vigano, and David Basin.
Constraint differentiation: Search-space reduction for the constraint-based analysis of security protocols.
Journal of Computer Security, 18(4):575–618, 2010.

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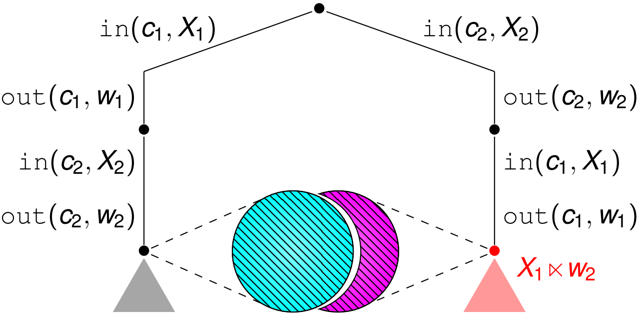

Dependency constraint: X_1 must depend on w_2.

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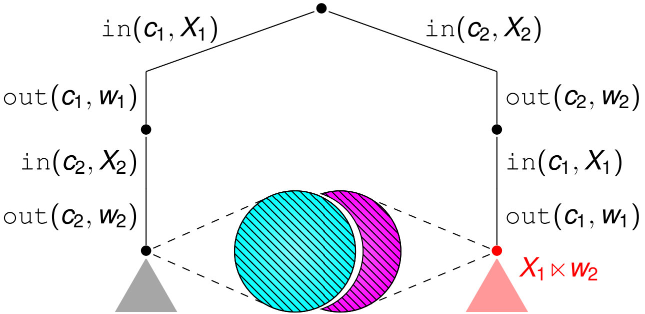

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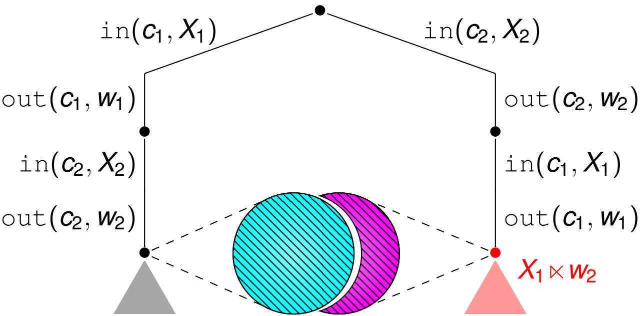

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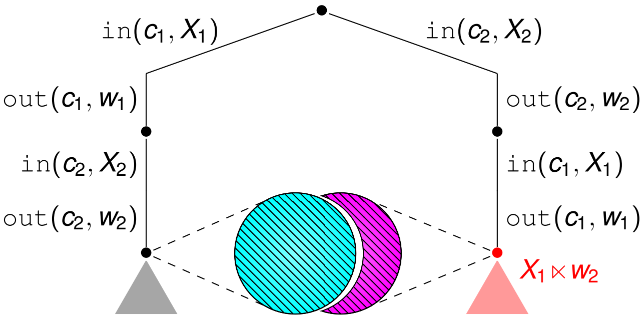

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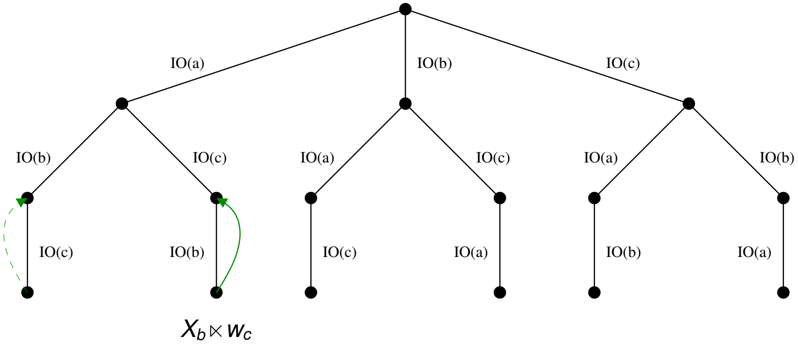
We can add constraints on the fly thanks to an order $<$.

- symmetry: Eliminate same traces on both sides
- Do not remove too much information (intruder can observe the order).

$$P = IO(a)|IO(b)|IO(c) \ \ \text{where} \quad IO(l) = \mathtt{in}(c_l, X_l).\mathtt{out}(c_l, w_l)$$



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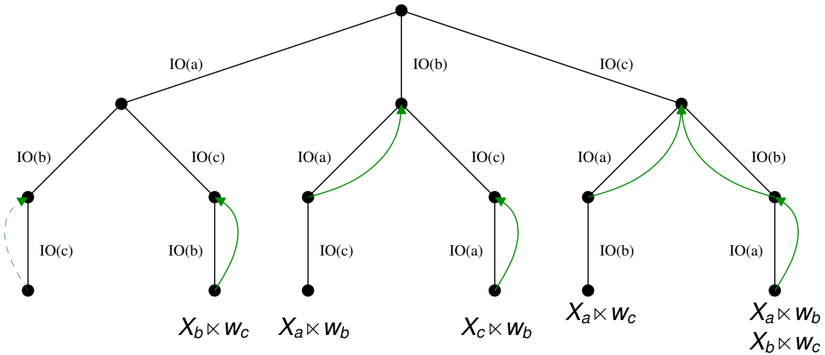

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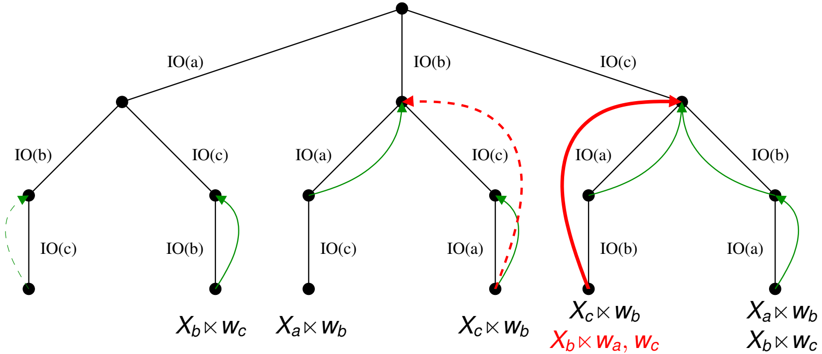

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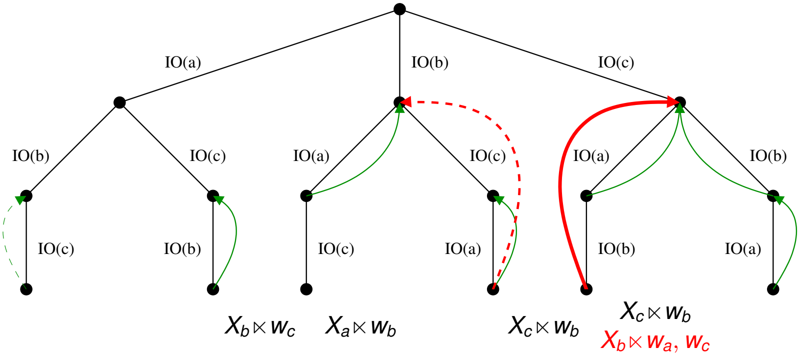

A block on c is executed following t, one input of the block must depend on one output of $\mathrm{dep}\,(t, c) = \{w_1 \ldots w_n\}$ if

- $t = t_1.IO(c_1).IO(c_2)\ldots IO(c_n).IO(c)$
- $c < c_1$;
- $c_1, c_2, \ldots, c_n < c$

Reduced semantics

Reduced semantics $\mapsto_d$

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$$\frac{(\mathcal{P};\Phi;\varnothing) \xrightarrow{\mathsf{tr}}_d (\mathcal{P}';\Phi';\mathcal{S}') \quad (\mathcal{P}';\Phi';\mathcal{S}') \xrightarrow{\mathtt{io}_c(\overrightarrow{X},\overrightarrow{w})}_c (\mathcal{P}'';\Phi'';\mathcal{S}'')}{(\mathcal{P};\Phi;\varnothing) \xrightarrow{\mathsf{tr}\cdot\mathtt{io}_c(\overrightarrow{X},\overrightarrow{w})}_d (\mathcal{P}'';\Phi'';\mathcal{S}'' \cup \{\overrightarrow{X} \ltimes \mathrm{dep}\,(\mathsf{tr}, c)\})}$$

Two possible semantics for $\mathsf{Sol}(X \ltimes w)$:

- second order: w occurs in the recipe $X\Theta$
- first order: for all recipe R, if $R\Phi = (X\Theta)\Phi$ then w occurs in R

Reduced semantics

Reduced semantics $\mapsto_d$

Compressed, symbolic semantics $+$ dependency constraints built on the fly.

$$\frac{(\mathcal{P};\Phi;\varnothing) \xmapsto{\mathrm{tr}}_d (\mathcal{P}';\Phi';\mathcal{S}') \quad (\mathcal{P}';\Phi';\mathcal{S}') \xmapsto{\mathtt{io}_c(\vec{X},\vec{w})}_c (\mathcal{P}'';\Phi'';\mathcal{S}'')}{(\mathcal{P};\Phi;\varnothing) \xmapsto{\mathrm{tr}\cdot\mathtt{io}_c(\vec{X},\vec{w})}_d (\mathcal{P}'';\Phi'';\mathcal{S}'' \cup \{\vec{X} \ltimes \mathrm{dep}(\mathrm{tr},c)\})}$$

Two possible semantics for $\mathrm{Sol}(X \ltimes w)$:

- second order: w occurs in the recipe $X\Theta$
- first order: for all recipe R, if $R\Phi = (X\Theta)\Phi$ then w occurs in R

Theorem 2

$$\approx_d^2 = \approx_s \qquad \approx_d^1 = \approx_s$$

Idea of the proof

- $[t]$: set of traces modulo valid permutations;
- $\mathrm{Min}([t])$: lexico. minimum of the class.

Lemma 1

If P has an trace t then it has all traces of $[t]$.

Lemma 2

- If P has an trace t then it has a reduced trace $\mathrm{Min}(\mathrm{t})$;
- P has no other reduced trace in $[t]$.

Outline

Conclusion

- New optimization in two steps:
  - compression
  - reduction
- applied to trace equivalence checking
- potentially exponential speed up
- early implementation in SPEC and Apte

Benchmarks

Tool	Protocol	Size	Ref (s)	Comp (s)	Red (s)
APTE	PA 1 Sess.	2/9/5	0.164	0.012	0.004
	PA 2 Sess.	4/15/5	> 237h	16.72	11.856
	PA 3 Sess.	6/21/5	> 237h	379696	91266
	BAC 1 S./1	4/52/6	13.98	0.02	0.008
	Simple 3 par	3/6/2	0.060	0.004	0.0040
	Simple 5 par	5/10/2	178.8	0.124	0.024
	Simple 7 par	7/14/2	> 163h	8.512	0.196
	Simple 10 par	7/14/2	> 163h	664	1.05
	Complex 4 par	4/10/4	99.87	0.55	0.136
	Complex 7 par	7/16/4	> 163h	198077	363.08
SPEC	2 par	2/22/10	$>$ 20 hours	13853.04	122.27
	7 par	7/14/2	$>$ 20 hours	13853	370.65
	WMF 1S	3/16/3	65.20	8.01	8.09
	WMF 2S $\perp$	6/24/3	7742.24	3.21	3.30

Future Work

- study constraint solving in more details
- study others redundancies $\rightsquigarrow$ recognize symmetries ?
- using those optimizations for interactive proofs of trace equivalence
- dealing with ! and nested parallels
- investigate such optimizations without the determinacy assumption

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Any question?

Outline

Informal Analogy: Focusing - Compression

- Compression: complete (wrt. equivalence) reduction of search space
- Focusing: complete (wrt. provability) reduction of search space

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Processes	LL formulae	polarity
$\mathtt{in}(c,x).P$	$\exists x.A$	synchronous
$\mathtt{out}(c,t).P$	$\forall x.A$	asynchronous
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$!^{a,\vec{c}}\,P$	$?P$	asynchronous

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compressed execution	focused derivation
completeness of $\approx_c$	completeness of focused proof system

Focused semantics

Focused execution: alternation of two phases

1. *Asynchronous phase:*
2. *Synchronous phase:*

Focused semantics

Focused execution: alternation of two phases

1. *Asynchronous phase:* **When:** $\exists$ output process.
   What: Only output actions are available.
2. *Synchronous phase:*

Focused semantics

Focused execution: alternation of two phases

1. *Asynchronous phase:* **When:** $\exists$ output process.
   What: Only output actions are available.
2. *Synchronous phase:* **When:** all processes start with an input or !.
   What: Choose one input process (or replicate one !): its is now under focus. Force to perform all its inputs until it reveals an asynchronous action.

Results (work in progress)

Even more effective compression handling REPLICATION and nested parallel compositions for determinate processes.

$$\text{if } A \xrightarrow{t_r} A_1 \text{ and } A \xrightarrow{t_r} A_2 \text{ then } A_1 = A_2.$$

Proof of completeness following the informal analogy and:

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From proofs to focused proofs: a modular proof of focalization in linear logic.
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- Strictly better: does the same for simple processes.
- Very modular: can be applied to any π-calculus-like.