Complexité avancée - TD 2

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Exercise 0: Warm up

Show that the following problems are NL-complete:

1. Deciding if a non-deterministic automaton $\mathcal{A}$ accepts a word w.
2. Deciding if a directed graph is strongly connected is NL-complete.
3. Deciding if a directed graph has a cycle.

Exercise 1: Dyck's language

- Let A be the language of balanced parentheses – that is the language generated by the grammar $S \rightarrow (S)|SS|\epsilon$. Show that $A \in \mathbf{L}$.
- What about the language B of balanced parentheses of two types? that is the language generated by the grammar $S \rightarrow (S)|[S]|SS|\epsilon$

Exercise 2: Restrictions of the SAT problem

1. Let 3-SAT be the restriction of SAT to clauses consisting of at most three literals (called 3-clauses). In other words, the input is a finite set S of 3-clauses, and the question is whether S is satisfiable. Show that 3-SAT is NP-complete for logspace reductions (assuming SAT is).
2. Let 2-SAT be the restriction of SAT to clauses consisting of at most two literals (called 2-clauses). Show that 2-SAT is in $\mathbf{P}$, using proofs by resolution.
3. Show that the complement of 2-SAT (i.e, the unsatisfiability of a set of 2-clauses) is NL-complete.
4. Conclude that 2-SAT is NL-complete.

Exercise 3: NL alternative definition

A Turing machine with *certificate tape* is a deterministic Turing machine with an extra read-only input tape called *the certificate tape*, which moreover is *read once* (*i.e.* the head on that tape can either remain on the same cell or move right, but never move left).

Define NL_{certif} to be the class of languages L such that there exists a polynomial $p : \mathbb{N} \rightarrow \mathbb{N}$ and a Turing machine with certificate tape M that runs in logarithmic space such that:

$$x \in L \text{ iff } \exists u, |u| \leq p(|x|) \text{ and } M \text{ accepts on input } (x, u)$$

1. Show that $NL_{certif} = NL$
2. What complexity class do you obtain if you remove the read-only constraint in the definition of a machine with certification tape ?

Exercise 4: smaller than L

What are the languages recognizable in constant space ?

Exercise 5: space hierarchy theorem

Using a diagonal argument, prove that for two space-constructible functions f and g such that $f(n) = o(g(n))$ (and as always $f, g \geq log$) we have $SPACE(f(n)) \subsetneq SPACE(g(n))$.

Exercise 6: Polylogarithmic space

Let PolyL $= \cup_{k\in\mathbb{N}} SPACE(log^k(n))$. Show that PolyL $\neq$ P.

Exercise 7: Closure under complement of context-sensitive languages

Recall that a context-senstitive language is a language generated by a context-sensitive grammar, that is a grammar $G = (V, \Sigma, P, S)$ where:

- V is a finite alphabet of non-terminal symbols;
- Σ is a finite alphabet of terminal symbols;
- $P \subset (V \cup \Sigma)^* \times (V \cup \Sigma)^*$ is a finite set of production rules, with the restriction that rules are of the form:

$$uXv \to uxv$$

  with $u, v \in (V \cup \Sigma)^*$, $x \in (V \cup \Sigma)^+$ and $X \in V$;
- $S \in V$ is the axiom.

A *linear bounded automaton* (LBA) is a single-tape, nondeterministic Turing machine where the input is written between special end-markers and the computation can never leave the space between these markers (nor overwrite them). Thus, over input $x_1, \ldots x_n$ the initial content of that tape is $\$x_1x_2\ldots x_n\#$ and the tape head can never leave this block of consecutive cells.

1. Show that a language is context-sensitive if and only if it is the language accepted by an LBA.
2. Show that a language is accepted by an LBA if and only if it is in **NSPACE**$(O(n))$.
3. Prove that context-sensitive languages are closed under complement (i.e. the complement of a context-sensitive language is context-sensitive).