Complexité avancée - TD 11

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Exercise 1: how big is PP?

Show that:

1. PP has complete problems.
2. $\mathsf{BPP} \subseteq \mathsf{PP}$
3. $\mathsf{NP} \subseteq \mathsf{PP}$
4. $MA \subseteq \mathsf{PP}$

Exercise 2: Polynomial Identity Testing

From previous exercise sheet.

Exercise 3: Counting versus Deciding

Define FP to be the class of functions $\Sigma^* \to \Sigma^*$ that are computable by a Turing machine working in polynomial time. Remember the definition of $\#\mathsf{P}$ and PP from this year's homework assignement.

Prove that $\mathsf{PP} = \mathsf{P}$ iff $\#\mathsf{P} = \mathsf{FP}$.

Exercise 4: how big is $\mathsf{P/poly}$?

1. Show that $\mathsf{P/poly} = \mathsf{P}^{\mathsf{SPARSE}}$, where SPARSE is the class of all sparse languages. A language L is said to be spare if there is a polynomial p such that for all $n \in \mathbb{N}$, $|L \cap \Sigma^n| \leq p(n)$.
   Hint: show $\mathsf{P/poly} \subseteq \mathsf{P}^{\mathsf{SPARSE}} \subseteq \mathsf{P}^{\mathsf{P}}/\mathsf{poly} \subseteq \mathsf{P/poly}$.
2. Show that $\mathsf{P/poly} = \mathsf{P}^{\mathsf{UNARY}}$, where UNARY is the class of all unary languages.
3. Deduce that $\mathsf{EXPTIME} = \mathsf{EXPSPACE}$ iff $\mathsf{PSPACE} \cap \mathsf{P/poly} = \mathsf{P}$.
4. Deduce that if $\mathsf{P} \neq \mathsf{RP}$, then $\mathsf{EXPTIME} \neq \mathsf{EXPSPACE}$.

Exercise 5: P versus RP

Define a random language A as a language such that for every word $w \in \Sigma^*$, $w \in A$ with probability 1/2.

Show that with probability 1 over the choice of a random A, $\mathsf{P}^A = \mathsf{RP}^A$.

Hint: use the random language as a random bit generator.